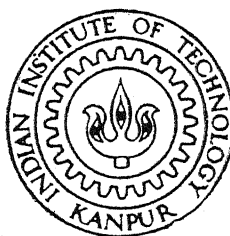


COMPUTER - AIDED SYNTHESIS AND ANIMATION OF PLANAR FOUR-LINK MECHANISMS

by
NITEESH KUMAR DUBEY



DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

FEBRUARY, 1998

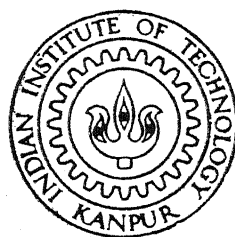
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COMPUTER-AIDED SYNTHESIS AND ANIMATION OF PLANAR FOUR-LINK MECHANISMS

A Thesis Submitted

in Partial Fulfillment of the Requirements

for the Degree of

Master of Technology

by

NITEESH KUMAR DUBEY



to the

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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CERTIFICATE

This is to certify that the thesis , entitled , “**COMPUTER-AIDED SYNTHESIS AND ANIMATION OF PLANAR FOUR-LINK MECHANISMS** ” by **Niteesh Kumar Dubey** is a record of work carried out under my supervision and has not been submitted elsewhere for a degree .

**Dr. A. K. MALLIK**

Professor

Department of Mechanical Engineering
Indian Institute of Technology , Kanpur .

Feb , 1998

Dedicated to
my
beloved and holy mother.

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Niteesh Kumar Dubey

ABSTRACT

Like most design problems , kinematic synthesis of planar linkages involves a number of trials to reach an acceptable or an optimal solution . Both graphical and analytical methods are available for the synthesis of four-link planar mechanisms . Independent of the method used , a solution arrived at always needs to be verified through animation . Consequently , a versatile software incorporating graphical and analytical methods and capable of analysis , synthesis and animation will be very useful to a mechanism designer .

This thesis , as a part of a D. S. T. (Department of Science and Technology) sponsored project , attempts to develop some specific problems , the solution of which require special graphical techniques or overall optimization problem . The Borland C++ language with graphic facility has been used because of its certain special features , which are eminently suitable for the problems under consideration . The software informs the user the kind of problems it can handle , asks for the input data (including designer ' s choice) , verifies the feasibility of the existence of solution , obtains the optimal solution and demonstrates the performance of the obtained solution .

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NOMENCLATURE

H	=	Stroke-length of the slider of a slider-crank mechanism
Q_{RR}	=	Quick-return ratio of a slider-crank mechanism
θ_i	=	Angle of rotation of the i^{th} link of a four-link mechanism
θ_2^*	=	Swing angle of the crank (CCW) when the rocker (or slider of a slider-crank mechanism) goes from outer dead-center to inner dead-center .
θ_4^*	=	Rocker swing angle corresponding to θ_2^*
θ_{24}^*	=	($\theta_2^* - \theta_4^*$)
E	=	The offset a slider-crank mechanism
L_i	=	Link length of the i^{th} link of a four-link mechanism
R, γ	=	The parameters , used to define a coupler point on the coupler of (or the connecting rod of a 3R-1P mechanism) a 4R mechanism
μ	=	Transmission angle of a four-link mechanism
θ_{2c}	=	Swing angle of the crank (CCW) when a coupler point of a crank-rocker mechanism goes from one cusp to another .
θ_{4c}	=	Swing angle of the rocker (CCW) corresponding to θ_{2c}
ω_i	=	Angular velocity of the i^{th} link of a four-link mechanism
α_i	=	Angular acceleration of the i^{th} link of a four-link mechanism

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1. INTRODUCTION

1.1 INTRODUCTION

Planar linkages are extensively used in various machineries in the field of agriculture, printing, textile and many others. A successful design of such a machinery very often requires, as a first step, an optimal kinematic design of various planar linkages used in it.

The kinematic design of a linkage starts with the number synthesis, i.e., with the decision on the number of links and kinematic pairs to be used for attaining the desired objective. However, a well-developed theory exists only for designing four-link mechanisms. The theory also points out, in some cases, the limitations of such four-link mechanisms. Only when the job in hand can not be achieved by a four-link mechanism, one needs to go for more complicated (e.g., six-link or eight-link) mechanisms. Even for designing such higher order linkages, very often the theory of four-link mechanisms serves as the starting point.

Over the last hundred years, various graphical and analytical techniques have been developed for the dimensional synthesis (i.e., the determination of the actual kinematic dimensions) of four-link planar mechanisms. The suitable choice of a method depends on the type of problem. In a four-link mechanism, the links connected to the fixed link (ground or frame of reference) are referred to as the input and output links, whereas the link not directly connected to the fixed link is called the floating link (e.g., the coupler, the connecting rod).

Depending on the required kinematic characteristics to be satisfied by the designed linkage, dimensional synthesis problems can be broadly classified as

[i] Motion Generation - It is defined as the control of a *line segment* in a plane (on the floating link) such that it assumes some prescribed set of sequential positions. The guidance may or may not be coordinated with the input motion.

[ii] Path Generation - If a tracer point on the floating link has to traverse through some specified points (on a specified path), then such a problem is classified as a problem of path generation. The generation of a prescribed path may or may not be coordinated with the input motion. No attempt is made to control the orientation of the link which contains the tracer point.

[iii] Function Generation - In this problem, the movements of the output and input links are coordinated according to a prescribed function [say, $y = f(x)$]. The change in the input movement corresponds to the change in the independent variable and that in the output movement to the change in the dependent variable. A function generator is conceptually a “ black box ” which delivers some predictable output in response to a known input.

[iv] Dead-Center Problems - These types of problems generally occur where the quick-return action of the output link for a constant input speed becomes important. Here, the linkage dimensions are determined for fulfilling the prescribed dead-center configurations.

In a real-life problem of path generation or function generation, it is generally not possible to design a mechanism which achieve the specified characteristics over the desired range of movements. One can only attempt for an approximate solution. There are two approaches to achieve an acceptable level of approximation.

[a] Accuracy Point Approach - In this approach, the generated characteristics of the designed mechanism exactly match with the desired characteristics only at some discrete positions within the specified range of movement. But at remaining configurations there exists some difference between the desired and generated characteristics which is called the *structural error*. One should select those precision points within the specified range of movement for which the maximum structural error becomes as small as possible. As a first approximation, *Chebyshev's accuracy points* ^[1] are generally chosen. With this approach, we can get a better solution if we take more number of precision points within the specified range of movement, but the number of precision points is limited by the number of available design parameters.

[b] Optimization Approach - In contrast to the above approach, here, the structural error, which may not be zero anywhere, is minimized in an overall sense in the entire range. It should be noted that, one can choose as many design positions as he / she wants. The number of design positions is independent of the number of available design parameters.

The accuracy point approach can be adopted while applying either a graphical or an analytical method, but the optimization approach is amenable to only analytical methods.

Finally, independent of the approach or technique adopted, a designed linkage has always to be checked for branch and order defects by checking through the entire range of movement. Moreover, there are always some assumed parameters and an optimal solution can be attained only after a number of trials.

All the points mentioned above make it amply clear that the dimensional synthesis of four-link mechanisms is an ideal problem to be tackled by a computer software with animation and graphic display. Commercial softwares are, of course, available, which are mostly suitable for kinematic and dynamic analysis. In this thesis, an attempt has been made, as a part of a D.S.T. (Department of Science and Technology) sponsored project, to develop a software which encompasses certain special graphical methods and optimisation approaches to solve a variety of dimensional synthesis problems of four-link planar mechanisms.

1.2 OBJECTIVE AND SCOPE OF THE PRESENT WORK

The objective of this work is to develop a computer software which can synthesize and animate four-link mechanisms for different types of problems, which are

- *Dead-center problems for crank-slider and crank-rocker mechanisms* : Here
 - [i] the quick-return ratio,
 - [ii] the stroke-length (in the case of 3R-1P) or the rocker swing angle (in the case of crank-rocker) and
 - [iii] one of the link lengths or offset

have to be given as inputs. This kind of problem has to be solved while designing quick-return mechanisms.

- *Generation of coupler curves* : To draw a segmented coupler curve corresponding to a specified coupler point on the floating link. Each segment corresponds to a specified crank-rotation. Thus the velocity of the coupler point at different positions can be judged by observing the length of each segment. Larger the length, greater is the velocity of the coupler point at that position.
- *Path generation by optimization approach* : In this case, the coordinates of several points on the specified path, coordinated with input angles will be required as inputs. In some cases, where the shape of the path along with the structural errors at some specified points are important, this approach is usually necessary.
- *Function generation by optimization approach* : Here, either a function or a set of coordinated input-output angles are prescribed. This is generally used in the synthesis of function generators.
- *Two cusps problems* : Here, a crank-rocker has to be designed so that a coupler point curve has two cusps at prescribed locations. Further, the crank rotation as the coupler point moves from one cusp to another is also prescribed. This type of problem arises in several processes (e.g. in the stamping and feeding processes, where a coupler point has to reach at its destination and return back without impact and shock, noiseless rocker-stop used in typewriters, recording machine in which a recording pencil writes only between two limiting points etc.).
- *To find the coupler curve cognates* : In the case of a path generation problem, a designed mechanism may not be accepted if its fixed pivots are located outside the available space. Then, in such a situation, its cognate generating the same coupler curve may be accepted. Similarly, if the initially designed mechanism turns out to be a Grashofian double-rocker, then its crank-rocker cognate may be an acceptable design.

- *Problems with specified kinematic variables (angular velocities and accelerations)* : To design a 4R linkage so that at a particular configuration, the prescribed values of
 - [i] angular displacements, velocities and accelerations of the input and output links or
 - [ii] angular velocities and accelerations of the input, floating and output links are achieved.

To meet the computational and animation requirements for tackling the above classes of problems, we have selected C⁺⁺ (Borland, with graphics environment) as the programming language because of its special characteristics (e.g. classes, objects, inheritance, overloading operator, data hiding etc.). The classes and some special member functions used in the present work are listed in the appendix.

2. DEAD - CENTER PROBLEMS

2.1 SLIDER - CRANK MECHANISM

Let us first consider the problem of synthesizing a slider-crank mechanism with prescribed stroke-length and quick-return ratio.

Referring to Fig 2.1, suppose

B_1 and B_2 represent the extreme positions of the slider 4 (when the crank 2 and the connecting rod 3 become collinear),

H = stroke-length of the slider = B_1B_2 ,

Q_{RR} = quick-return ratio,

θ_2^* = An angle through which the crank rotates when
the slider goes from B_1 to B_2

or $\theta_2^* = 2\pi \{ (Q_{RR}) / (1 + Q_{RR}) \}$,

$\theta_2^{**} = \angle B_1O_2B_2 = (\theta_2^* - \pi)$

and $\beta = \angle O_2B_1B_2$.

If only H and Q_{RR} (or θ_2^{**}) are given, then we can have infinite solutions. It can be understood easily by using the property of a circle which states that “ a chord of a circle subtends equal angle at all the circumferential points which are on the same side of the chord ”. So O_2 can be placed anywhere on the circumference of a circle (CR_2) passing through B_1 , B_2 and O_2 . Thus, for a unique design, besides H and θ_2^* (i.e., the quick-return ratio), one more kinematic dimension, viz., L_2 (crank length) or L_3 (connecting rod length) or E (offset), must be either prescribed or assumed. Depending on which one of the three dimensions is given (or assumed), three methods of determining the other two are discussed below.

Case 1. H , θ_2^* and E are prescribed, L_2 and L_3 are to be determined :

From Δ 's O_2NB_1 and O_2NB_2 ,

$$E = (L_3 - L_2) \sin(\beta + \theta_2^{**}) = (L_3 + L_2) \sin(\beta) \quad [2.1.1]$$

and

$$H = (L_3 + L_2) \cos(\beta) - (L_3 - L_2) \cos(\beta + \theta_2^{**}). \quad [2.1.2]$$

From Equations 2.1.1 and 2.1.2,

$$\sin(\beta + \theta_2^{**}) \sin(\beta) = (E/H) \sin(\beta + \theta_2^{**} - \beta)$$

or

$$\cos(2\beta + \theta_2^{**}) = \cos(\theta_2^{**}) - (2E/H) \sin(\beta + \theta_2^{**} - \beta)$$

or

$$\beta = [\cos^{-1}\{\cos(\theta_2^{**}) - (2E/H) \sin(\theta_2^{**})\} - \theta_2^{**}] / 2. \quad [2.1.3]$$

Now

$$O_2B_1 = (L_3 + L_2) = E / \sin(\beta) \quad [2.1.4]$$

and

$$O_2B_2 = (L_3 - L_2) = E / \sin(\beta + \theta_2^{**}). \quad [2.1.5]$$

Therefore,

$$L_3 = (O_2B_1 + O_2B_2) / 2 \quad \text{and} \quad L_2 = (O_2B_1 - O_2B_2) / 2. \quad [2.1.6]$$

Case 2. H , θ_2^* and L_3 are prescribed, L_2 and E are to be determined :

From $\Delta O_2B_2B_1$,

$$\cos(\theta_2^{**}) = \{ (L_3 + L_2)^2 + (L_3 - L_2)^2 - H^2 \} / \{ 2(L_3 + L_2)(L_3 - L_2) \}$$

or

$$\cos(\theta_2^{**}) = \{ L_3^2 + L_2^2 - (H^2 / 2) \} / (L_3^2 - L_2^2), \quad [2.1.7]$$

which gives

$$L_2 = \sqrt{ \left[(H^2 / 2) - L_3^2 \{ 1 - \cos(\theta_2^{**}) \} \right] / \{ 1 + \cos(\theta_2^{**}) \} }. \quad [2.1.8]$$

From Equation 2.1.1, we get

$$\begin{aligned} (L_3 - L_2) \{ \sin(\beta) \cos(\theta_2^{**}) + \cos(\beta) \sin(\theta_2^{**}) \} \\ = (L_3 + L_2) \sin(\beta) \end{aligned}$$

or

$$\tan(\beta) = \{ (L_3 - L_2) \sin(\theta_2^{**}) \} / \{ (L_3 + L_2) - (L_3 - L_2) \cos(\theta_2^{**}) \}$$

or

$$\beta = \tan^{-1} [\{ (L_3 - L_2) \sin(\theta_2^{**}) \} / \{ (L_3 + L_2) - (L_3 - L_2) \cos(\theta_2^{**}) \}]. \quad [2.1.9]$$

Hence

$$E = (L_3 + L_2) \sin(\beta). \quad [2.1.10]$$

Case 3. H, θ_2^* and L_2 are prescribed, L_3 and E are to be determined :

From Equation 2.1.7,

$$L_3 = \sqrt{ [(H^2/2) - L_2^2 \{ 1 + \cos(\theta_2^{**}) \}] / \{ 1 - \cos(\theta_2^{**}) \} }. \quad [2.1.11]$$

Then using Equations 2.1.9 and 2.1.10 we can get the offset E.

Optimal solution : As we have seen above, that besides H and Q_{RR} (or θ_2^*), one has to give either L_2 or L_3 or E to obtain a unique solution for a 3R-1P mechanism. If only H and Q_{RR} (or θ_2^*) are available then one can also design a unique slider-crank mechanism for which the minimum transmission angle is maximum.

For smooth functioning of a mechanism in real life, we should ensure that, besides satisfying the kinematic requirements, the mechanism moves freely. Neglecting the effects of the gravity, inertia and friction all binary links become two force members. Consequently, the force in any such link is along the link vector and the force analysis becomes simple. To judge the free running quality of a mechanism, the transmission angle is considered as one of the most important parameter.

For a slider-crank mechanism the *transmission angle* is defined as the absolute acute angle between the connecting rod and the normal to the slider movement (Fig - 2.2). The minimum transmission angle ^[1] for this mechanism is given by

$$\mu_{\min} = \cos^{-1} [(L_2 + E) / L_3]. \quad [2.1.12]$$

Referring to Fig 2.1,

from $\Delta O_2B_1B_2$, we observe that

$$L_3 - L_2 = \{ H / \sin (\theta_2^{**}) \} \sin (\beta)$$

and

$$L_3 + L_2 = \{ H / \sin (\theta_2^{**}) \} \sin (\beta + \theta_2^{**}).$$

Therefore,

$$L_2 = \left[\frac{H}{2 \sin (\theta_2^{**})} \right] \{ \sin (\beta + \theta_2^{**}) - \sin (\beta) \} \quad [2.1.13]$$

and

$$L_3 = \left[\frac{H}{2 \sin (\theta_2^{**})} \right] \{ \sin (\beta + \theta_2^{**}) + \sin (\beta) \}. \quad [2.1.14]$$

Again, from ΔO_2B_1N , we get

$$E = (L_3 + L_2) \sin (\beta)$$

or

$$E = \left[\frac{H}{\sin (\theta_2^{**})} \right] \sin (\beta + \theta_2^{**}) \sin (\beta). \quad [2.1.15]$$

To maximize the minimum transmission angle μ_{\min} (given by Equation 2.1.12), one must minimize the value of $[(L_2 + E) / L_3]$ which is a function of the variable β . Using Equations 2.1.13, 2.1.14 and 2.1.15, we get

$$\frac{L_2 + E}{L_3} = 1 - \left[\frac{2 \sin (\beta) \{ 1 - \sin (\beta + \theta_2^{**}) \}}{\sin (\beta + \theta_2^{**}) + \sin (\beta)} \right]. \quad [2.1.16]$$

From Fig 2.1, it is obvious that O_2 can not be above the line passing through B_1 and B_2 ; and O_2B_1 must not be less than O_2B_2 (since B_1 and B_2 are the outer and inner dead-centers, respectively). This implies that β varies from 0 to $(\pi - \theta_2^{**}) / 2$.

Thus, we have a function $[(L_2 + E) / L_3]$ of a single variable β . The variable β varies from 0 to $(\pi - \theta_2^{**}) / 2$. As we know that, to minimize a single variable function for a given range of the variable, one can use the *golden section search method* ^[2]. For the present problem, using this optimization method, we get β_{opt} corresponding to the minimum value of $[(L_2 + E) / L_3]$ and hence L_2 , L_3 , E and μ_{min} (from Equations 2.1.13, 2.1.14, 2.1.15 and 2.1.12 respectively) are known.

Flow chart with illustrations - Using the computational flow chart (Chart 2.1), one can easily understand the steps used in the program for dead-center problems of a slider-crank mechanism. It also helps the designer to run the program. After computing the design parameters of the mechanism, the software itself checks the existence of the mechanism. If the mechanism exists, then `DRAW_3R1P()` can be used to animate the slider-crank mechanism. For verification of the results, one should opt for *dead-center configurations* which display all the dead-center configurations along with the stroke-length.

To illustrate the synthesis of a slider-crank mechanism (case 1), we have considered a problem with $H = 6.0$ unit, $Q_{RR} = 1.4$ and $E = 1.0$ unit.

For this problem, the final design yields

$$L_2 = 2.8629 \text{ unit,}$$

$$L_3 = 4.489 \text{ unit}$$

$$\text{and } \mu_{min} = 30.73^0.$$

To verify the result, one can see its dead-center configurations shown in Plot 2.1.

For an optimal design problem, taking $H = 6.0$ unit and $Q_{RR} = 1.4$, we design a slider-crank mechanism for which

$$L_2 = 2.7307 \text{ unit,}$$

$$L_3 = 5.5222 \text{ unit,}$$

$$E = 1.9198 \text{ unit}$$

$$\text{and } \mu_{\min} = 32.63^{\circ}.$$

Its dead-center configurations are shown in Plot 2.2.

2. 2 CRANK - ROCKER MECHANISM

Now let us consider the synthesis of a crank-rocker mechanism for dead-center problems. From Fig 2.3, $O_2A_1B_1O_4$ (shown in Fig 2.3) is a crank-rocker mechanism. The rocker O_4B_1 swings by an angle θ_4^* when the crank O_2A_1 rotates by θ_2^* (CCW) and correspondingly the moving hinge of the follower traverses from B_1 (outer-dead center) to B_2 (inner-dead center). The link lengths (L_1 , L_2 , L_3 and L_4) are indicated in the figure.

Here $B_1B_2 = 2 L_4 \sin (\theta_4^* / 2)$ and the offset E can be taken as the perpendicular distance of line B_1B_2 from O_2 . So if the length B_1B_2 (\equiv the stroke-length), Q_{RR} (or θ_2^*) and one of the lengths (L_1 or L_2 or L_3 or E) are given, then one can synthesize a crank-rocker mechanism in the same way as was done for the slider-crank mechanism.

But in real life, generally Q_{RR} (or θ_2^*), the rocker swing angle θ_4^* and the fixed length L_1 are given to synthesize a crank-rocker mechanism. In such a situation, the solution is obtained by the Alt's construction ^[1]. The Alt's construction gives infinite solutions, but one must select the mechanism for which the minimum transmission angle is maximum considering the free running quality of the mechanism.

The *transmission angle* of a 4R mechanism is defined as the absolute value of the acute angle between the output link and the coupler (Fig 2.4). Suppose a torque T_2 is applied at link 2, then the generated torque T_4 (on link 4) will be directly proportional to the force F_t [$= F \sin(\mu)$] where μ is the transmission angle and F is the produced axial force in the coupler. The radial component F_r provides only tension or compression in link 4. For good performance of a mechanism it is suggested that $\mu_{\min} > 30^{\circ} - 35^{\circ}$.

The transmission angle ^[1] μ (for the crank-rocker) is given by

$$\mu = \cos^{-1} [\{ (L_3^2 + L_4^2) - (L_1^2 + L_2^2) - 2 L_1 L_2 \cos \theta_2 \} / (2 L_3 L_4)]. \quad [2.2.1]$$

If $\theta_2^* \geq \pi$ [or $(L_3^2 + L_4^2) \geq (L_1^2 + L_2^2)$] then

$$\mu_{\min} = \cos^{-1} [\{ (L_3^2 + L_4^2) - (L_1^2 + L_2^2) + 2 L_1 L_2 \} / (2 L_3 L_4)] \quad [2.2.2]$$

and if $\theta_2^* \leq \pi$ [or $(L_3^2 + L_4^2) \leq (L_1^2 + L_2^2)$] then

$$\mu_{\min} = \pi - \cos^{-1} [\{ (L_3^2 + L_4^2) - (L_1^2 + L_2^2) - 2 L_1 L_2 \} / (2 L_3 L_4)]. \quad [2.2.3]$$

To get the optimum solution, the designer generally uses Volmer's nomogram (Plot 2.4). To design a crank-rocker mechanism for prescribed dead-center configurations on a computer, one must have to feed the data from Volmer's nomogram. But here, we have derived a single variable function (say μ_{\min}) which is again optimized by the *golden section search method* to obtain the corresponding unknown design parameters (L_2, L_3 and L_4). Finally we verified the results with the Volmer's nomogram.

Using the Alt's constructions, the unknown design parameters of a crank-rocker mechanism for various situations are presented below. In all the cases θ_2^* and θ_4^* are measured CCW from the outer to the inner dead-center configurations.

Let

$$\theta_2^{**} = \theta_2^* - \pi \quad \text{and} \quad \theta_{24}^* = \theta_2^* - \theta_4^*. \quad [2.2.4]$$

Case 1 : [$\theta_{24}^* < \pi$ and $\theta_2^* \geq \pi$]

Referring to Fig 2.5, O_2 and O_4 are the fixed pivots such that $O_2O_4 = L_1$. At O_2 , a line O_2E_2 is drawn at an angle of $(- \theta_2^* / 2)$ to O_2O_4 , and another line O_4E_4 is drawn through O_4 at an angle of $(- \theta_4^* / 2)$ to O_2O_4 . These two lines intersect at F and the circle CR_1 is drawn with O_2F as the diameter. The mid-normal of O_2F intersects O_4F at C_2 . Another circle CR_2 , is drawn with its center at C_2 and radius as C_2F .

In this case, since C_2F is greater than (if $\theta_2^* > \pi$) or equal to (if $\theta_2^* = \pi$) C_2O_4 . The circle CR_2 intersects O_2O_4 at S which coincides with O_4 (if $\theta_2^* = \pi$) or lies outside (if $\theta_2^* > \pi$) O_2O_4 (as shown in Fig 2.5).

A line O_4N is drawn through O_4 at an angle of $(- \theta_4^*)$ to O_2O_4 intersecting the circle CR_2 at the point N . The point B_1 can now be taken anywhere on the arc NS . The line O_2B_1 intersects CR_1 at A_1 . Thus, $O_2A_1B_1O_4$ is the required 4R crank-rocker linkage at its outer dead-center configuration. All the unknowns and some of the assumed symbols are shown in the figure.

Now to determine the unknown design parameters, referring to Fig 2.5,

let

$$C_1 \equiv (x_{c1}, y_{c1}), \quad r_1 = \text{the radius of circle } CR_1,$$

$$C_2 \equiv (x_{c2}, y_{c2}) \text{ and } r_2 = \text{the radius of circle } CR_2.$$

From ΔO_2FO_4

$$r_1 = O_2F / 2$$

or

$$r_1 = \{ L_1 \sin (\theta_4^* / 2) \} / \{ 2 \sin (\theta_{24}^* / 2) \} \quad [2.2.5]$$

$$[\because \angle O_2FO_4 = \theta_{24}^* / 2],$$

$$x_{c1} = - r_1 \cos (\theta_2^* / 2) \quad [2.2.6]$$

$$\text{and } y_{c1} = r_1 \sin (\theta_2^* / 2). \quad [2.2.7]$$

From ΔC_1FC_2

$$r_2 = FC_2 = r_1 \sec (\theta_{24}^* / 2), \quad [2.2.8]$$

from ΔO_2FO_4

$$x_{c2} = 2 x_{c1} + r_2 \cos (\theta_4^* / 2) \quad [2.2.9]$$

$$[\because \angle O_2O_4F = (\theta_4^*) / 2]$$

and

$$y_{c2} = 2 y_{c1} - r_2 \sin (\theta_4^* / 2). \quad [2.2.10]$$

Let

$$N \equiv (x_n, y_n) \text{ and } O_4N = l_n,$$

where

$$x_n = L_1 - l_n \cos(\theta_4^*) \quad [2.2.11]$$

$$\text{and } y_n = l_n \sin(\theta_4^*). \quad [2.2.12]$$

Since N is on the circle CR₂, we can write

$$(x_n - x_{c2})^2 + (y_n - y_{c2})^2 = r_2^2.$$

Putting the values of x_n and y_n in the above equation and on simplification, we get

$$l_n^2 + \{ 2(x_{c2} - L_1) \cos(\theta_4^*) - 2y_{c2} \sin(\theta_4^*) \} l_n + (L_1^2 - 2L_1 x_{c2}) = 0. \quad [2.2.13]$$

Equation 2.2.13 gives two values of l_n . Here, the positive value of l_n is chosen as y_n is positive. Then, from Equations 2.2.11 and 2.2.12, we can have x_n and y_n .

Now

$$\tan \beta_n = y_n / x_n$$

$$\text{or } \beta_n = \tan^{-1}(y_n / x_n). \quad [2.2.14]$$

$$\text{For the point S, } \beta_s = 0.$$

Thus, β can vary from β_s to β_n .

Now let us consider a general point B₁ on the circle CR₂ at an angle β (which may vary from β_s to β_n).

$$\text{Suppose } O_2B_1 = L = L_2 + L_3.$$

Therefore,

$$(L \cos \beta - x_{c2})^2 + (L \sin \beta - y_{c2})^2 = r_2^2$$

or

$$L = 2(x_{c2} \cos \beta + y_{c2} \sin \beta) \quad [\because x_{c2}^2 + y_{c2}^2 = r_2^2]. \quad [2.2.15]$$

Similarly the point A₁ lies on the circle CR₁ at the same angle β , therefore,

$$(L_2 \cos \beta - x_{c1})^2 + (L_2 \sin \beta - y_{c1})^2 = r_1^2$$

or

$$L_2 = 2 (x_{cl} \cos \beta + y_{cl} \sin \beta) \quad [\because x_{cl}^2 + y_{cl}^2 = r_1^2]. \quad [2.2.16]$$

Now

$$L_3 = L - L_2 \quad [2.2.17]$$

and from $\Delta O_2B_1O_4$

$$L_4 = (L^2 + L_1^2 - 2 L_1 L \cos \beta)^{1/2}. \quad [2.2.18]$$

Thus, for any angle β (varying from β_s to β_n) we can have the link lengths (L_1 , L_2 , L_3 and L_4) to design the crank-rocker mechanism. But we have already discussed that the mechanism for which μ_{\min} is maximum will be the best regarding the transmission quality. Now we have to maximize the minimum transmission angle (given by Equation 2.2.2)

$$\mu_{\min} = \cos^{-1} [\{ (L_3^2 + L_4^2) - (L_1^2 + L_2^2) + 2 L_1 L_2 \} / (2 L_3 L_4)]. \quad [2.2.19]$$

Since L_2 , L_3 and L_4 are functions of β , hence μ_{\min} is also a function of the same variable β (varying from β_s to β_n). To maximize μ_{\min} , we have used the *golden section search method*. As we know that this optimization method minimizes a single variable function in a given range of its variable. Therefore, in the present problem, we have chosen $(- \mu_{\min})$ (since μ_{\min} is to be maximized) as a function and β as a variable.

After optimization, we can get β_{opt} (corresponding to $(\mu_{\min})_{\text{max}}$) and hence the desired unknown link lengths (L_2 , L_3 and L_4).

Case 2 : [$\theta_{24}^* < \pi$ and $\theta_2^* < \pi$]

Referring to Fig 2.6, in this case, the point O_4 lies outside the circle CR_2 because $FC_2 < C_2O_4$. Hence, the line O_4N (at an angle - θ_4^*) cuts the circle CR_2 at two points both with positive y coordinates.

From Equation 2.2.13, one value of l_n corresponds to the point N and the other to

the point S. Therefore, using Equations 2.2.11, 2.2.12 and 2.2.14 we can have β_s and β_n . Further, from Equation 2.2.3,

$$\mu_{\min} = \pi - \cos^{-1} [\{ (L_3^2 + L_4^2) - (L_1^2 + L_2^2) - 2 L_1 L_2 \} / (2 L_3 L_4)]. \quad [2.2.20]$$

Other Equations remain the same as in case 1. Again using the same optimization method, μ_{\min} is maximized to get β_{opt} (corresponding to $(\mu_{\min})_{\text{max}}$) and hence the desired unknown link lengths (L_2, L_3 and L_4).

Case 3 : [$\theta_{24}^* = \pi$]

In this special situation (Fig 2.7), the point C_2 goes to infinity (since $\angle O_2FO_4 = \pi / 2$) and the circle CR_2 degenerates into the straight line O_2F .

It is obvious that, $O_2F = FN$ since $\angle O_2O_4F = \angle FO_4N = \theta_4^* / 2$. The point B_1 can now be located anywhere on the line O_2F above N and the crank length (= O_2F) remains the same for all the solutions.

From Fig 2.7, we observe that

$$\beta = \pi - (\theta_2^* / 2), \text{ which is constant,} \quad [2.2.21]$$

$$r_1 = L_1 \sin (\theta_4^* / 2) / 2 \quad [2.2.22]$$

$$\text{and } L_2 = 2 r_1 \text{ which is again constant.} \quad [2.2.23]$$

Though the point S, in this case, can be at infinity, to achieve a compact linkage O_2S is restricted to $4 O_2F$. Thus the variable L (= O_2B_1) is restricted in the range $2L_2$ (= O_2N) $< O_2B_1 < 4L_2$ (= O_2S). After optimization it has been found that it gives the same result as the *Volmer's nomogram*.

$$\text{Now } L_3 = L - L_2 \quad [2.2.24]$$

and from ΔO_4B_1D

$$L_4 = \{ (L \sin \beta)^2 + (L \cos \beta - L_1)^2 \}^{1/2}. \quad [2.2.25]$$

In this case,

$$\mu_{\min} = \cos^{-1} [\{ (L_3^2 + L_4^2) - (L_1^2 + L_2^2) + 2 L_1 L_2 \} / (2 L_3 L_4)]. \quad [2.2.26]$$

The *golden section search method* is again used to minimize $(-\mu_{\min})$, but here the variable is L which varies from $2L_2 (= L_N)$ to $4L_2 (= L_S)$. Thus, we get L_{opt} and hence L_3 and L_4 .

Case 4 : [$\theta_{24}^* > \pi$]

This case is also similar to case 1 except for the change of location of the center C_2 of the circle CR_2 and as in case 2, the point S lies above the line O_2O_4 (Fig 2.8).

Therefore,

$$r_2 = FC_2 = -r_1 \sec(\theta_{24}^* / 2) \quad [2.2.27]$$

and from ΔO_2FO_4

$$x_{c2} = 2 x_{c1} - r_2 \cos(\theta_4^* / 2) \quad [2.2.28]$$

and

$$y_{c2} = 2 y_{c1} + r_2 \sin(\theta_4^* / 2). \quad [2.2.29]$$

Again Equation 2.2.13 gives two values of l_n , one for the point N and the other for the point S . Then, from Equations 2.2.11, 2.2.12 and 2.2.14 we can have β_s and β_n .

Further, from Equation 2.2.2,

$$\mu_{\min} = \cos^{-1} [\{ (L_3^2 + L_4^2) - (L_1^2 + L_2^2) + 2 L_1 L_2 \} / (2 L_3 L_4)]. \quad [2.2.30]$$

Other Equations remain the same as in case 1. Then using the same optimization method μ_{\min} is maximized to get β_{opt} [corresponding to $(\mu_{\min})_{\text{max}}$] and hence the desired unknown link lengths $(L_2, L_3 \text{ and } L_4)$.

The following points regarding the Alt's construction ^[1] may be noted.

[i] If a point B_1 is taken outside the region NS, then the rocker has to cross the frame-line during its movement, which is not possible in crank-rocker mechanism.

[ii] For a given value of θ_4^* , the minimum and maximum values of θ_2^* (for a crank-rocker linkage to exist) are given, respectively, by

$$(\theta_2^*)_{\min} = (\pi + \theta_4^*) / 2 \quad \text{and} \quad (\theta_2^*)_{\max} = (3\pi + \theta_4^*) / 2.$$

[2.2.31]

In these limiting conditions, the line O_4N in Alt's construction becomes tangent to CR_2 .

Flow chart with an illustration - The steps used in the programming are shown in Chart 2.2. The restrictions (given by Equation 2.2.31) has been used to control the input data for achieving a feasible solution. Here, the role of the function GOLD (type, β_n , β_s , x_{C1} , y_{C1} , x_{C2} , y_{C2}), used in the *golden section search method* is very important. The first variable of the function selects the type (here, type = 1 for cases 1, 2, 4 and type = 2 for case 3) where as (β_n , β_s) define the range of the variable and the last four variables determine the transmission angle through another function MEU (type, x_{C1} , y_{C1} , x_{C2} , y_{C2} , β). In function GOLD (), a number τ (tau = 0.618) has been used, this is known as the golden number ^[2]. An accuracy controlling parameter ε has also been considered in the same function, which controls the accuracy of $(\mu_{\min})_{\max}$ and β_{opt} . We can get very accurate values just by controlling ε , which is really an advantage over the conventional nomogram method.

After knowing the link lengths, the function DRAW_4R() animates the mechanism. For verification of the results, one can use the option dead-center configuration.

Now as an illustration let us consider a dead-center problem for which

$$L_1 = 6.0 \text{ unit, } \theta_4^* = 60^\circ \text{ and } \theta_2^* = 210^\circ.$$

The obtained final results are

$$(\mu_{\min})_{\max} = 32.63^{\circ},$$

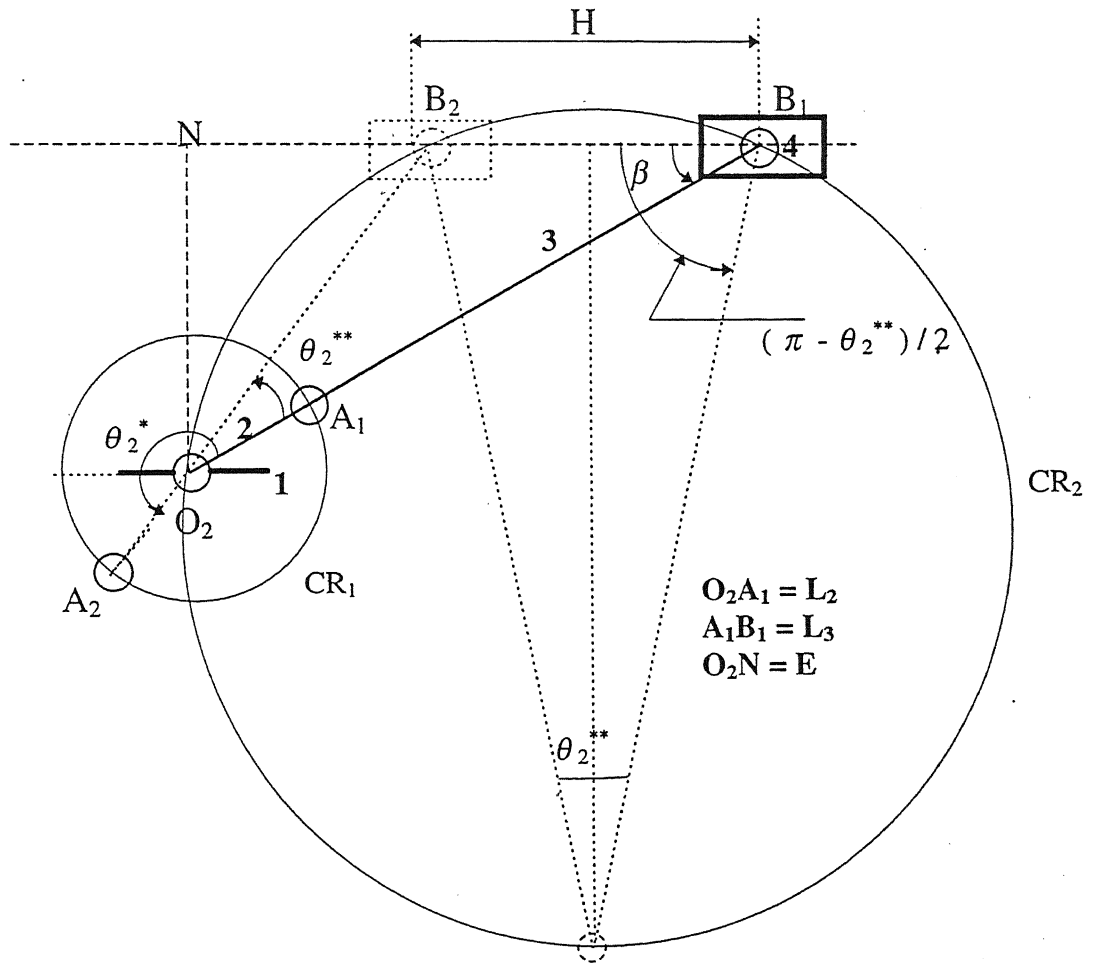
$$\beta_{\text{opt}} = 46.57^{\circ},$$

$$L_2 = 2.73 \text{ unit},$$

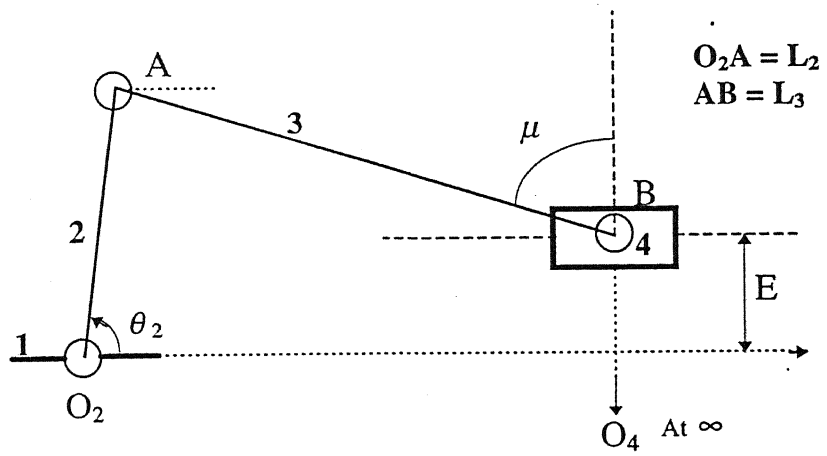
$$L_3 = 5.519 \text{ unit}$$

$$\text{and } L_4 = 6.0 \text{ unit}$$

The dead-center configurations of this mechanism are shown in Plot 2.3. One can also verify $(\mu_{\min})_{\max}$ and β_{opt} from the Volmer's nomogram (shown in Plot 2.4).

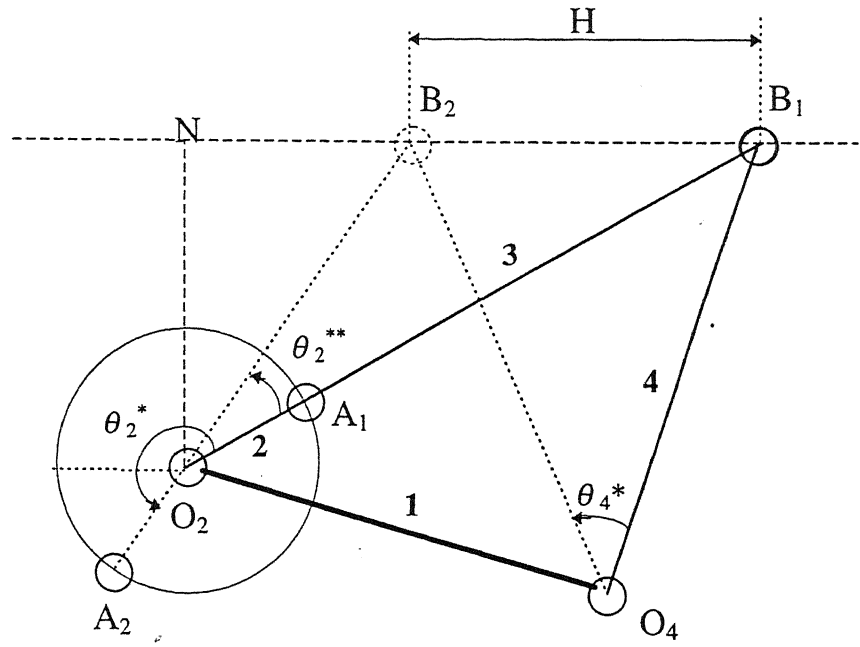


[Fig 2.1]



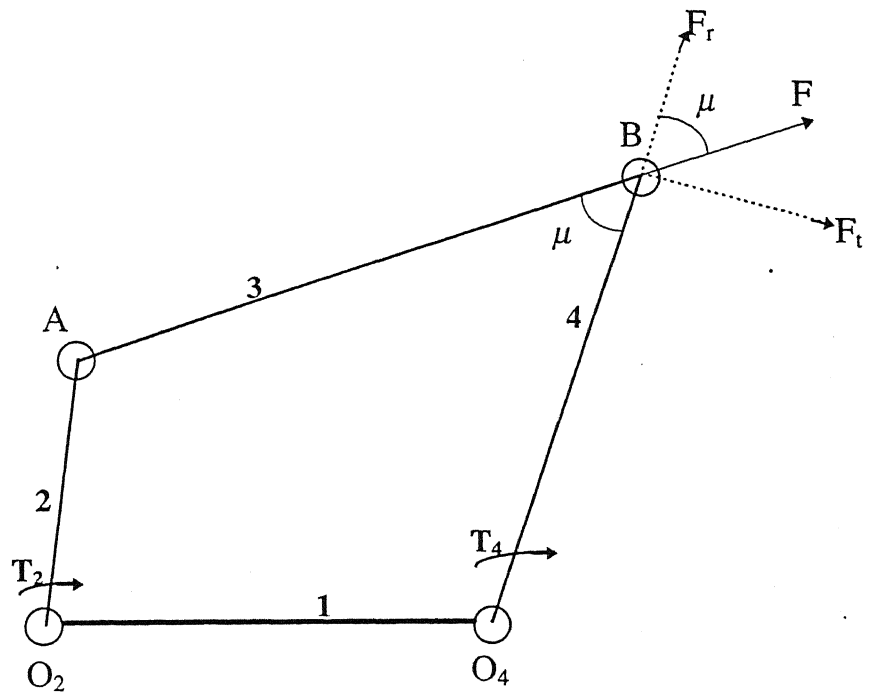
[Fig 2.2]

$$\begin{aligned} O_2O_4 &= L_1 \\ O_2A_1 &= L_2 \\ A_1B_1 &= L_3 \\ O_4B_1 &= L_4 \\ O_2N &= E \end{aligned}$$

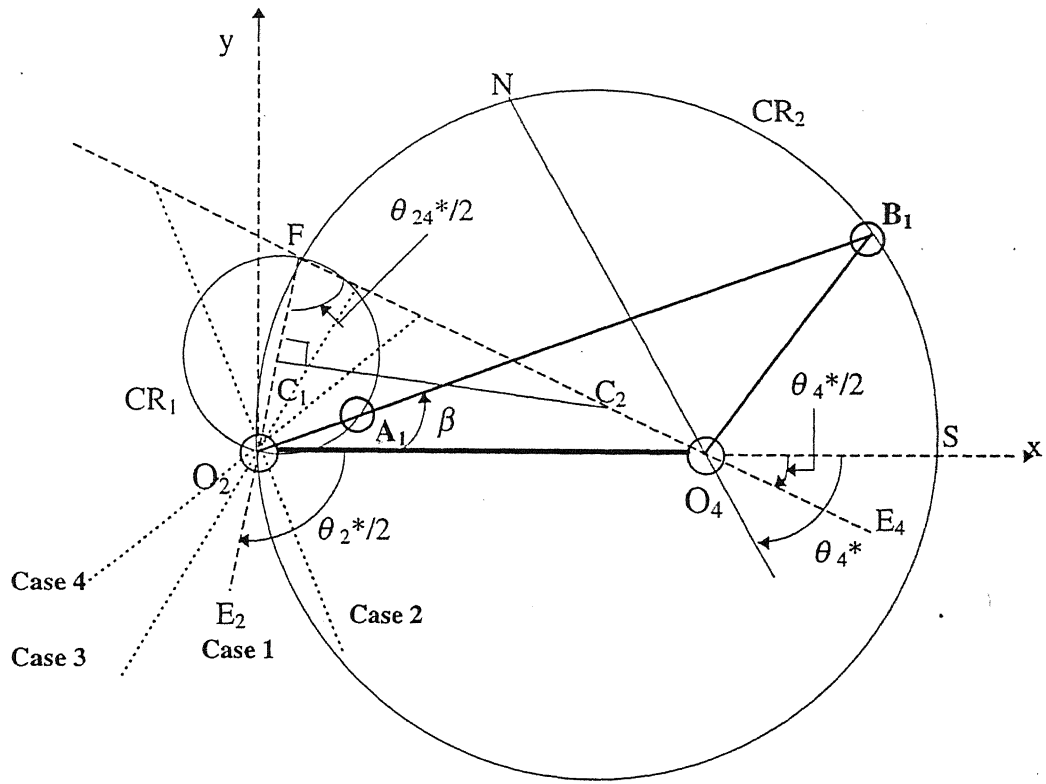


[Fig 2.3]

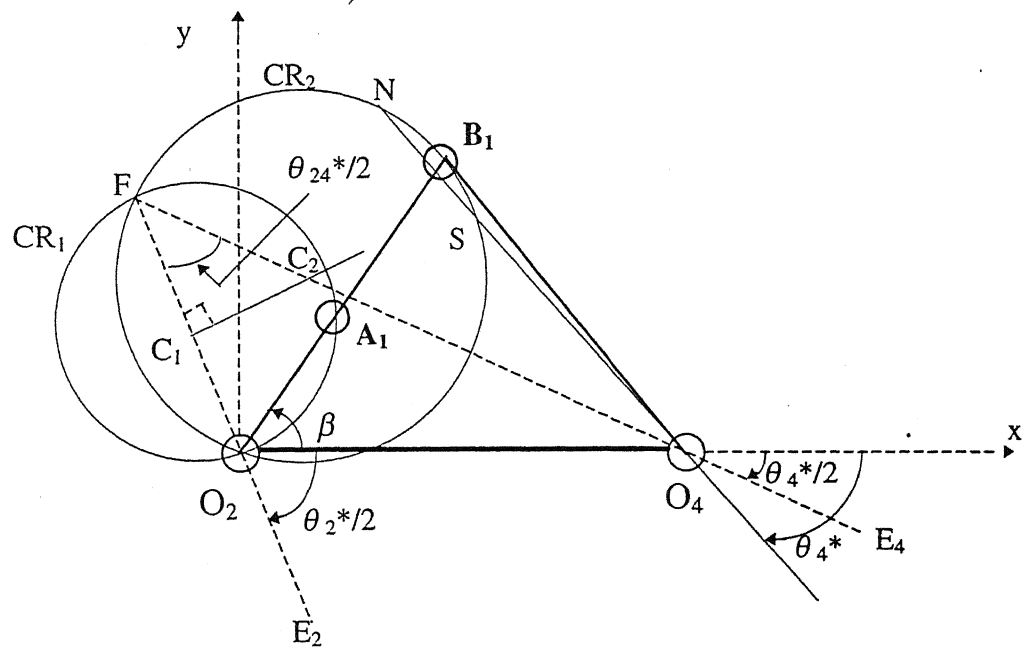
$$\begin{aligned} O_2O_4 &= L_1 \\ O_2A &= L_2 \\ AB &= L_3 \\ O_4B &= L_4 \end{aligned}$$



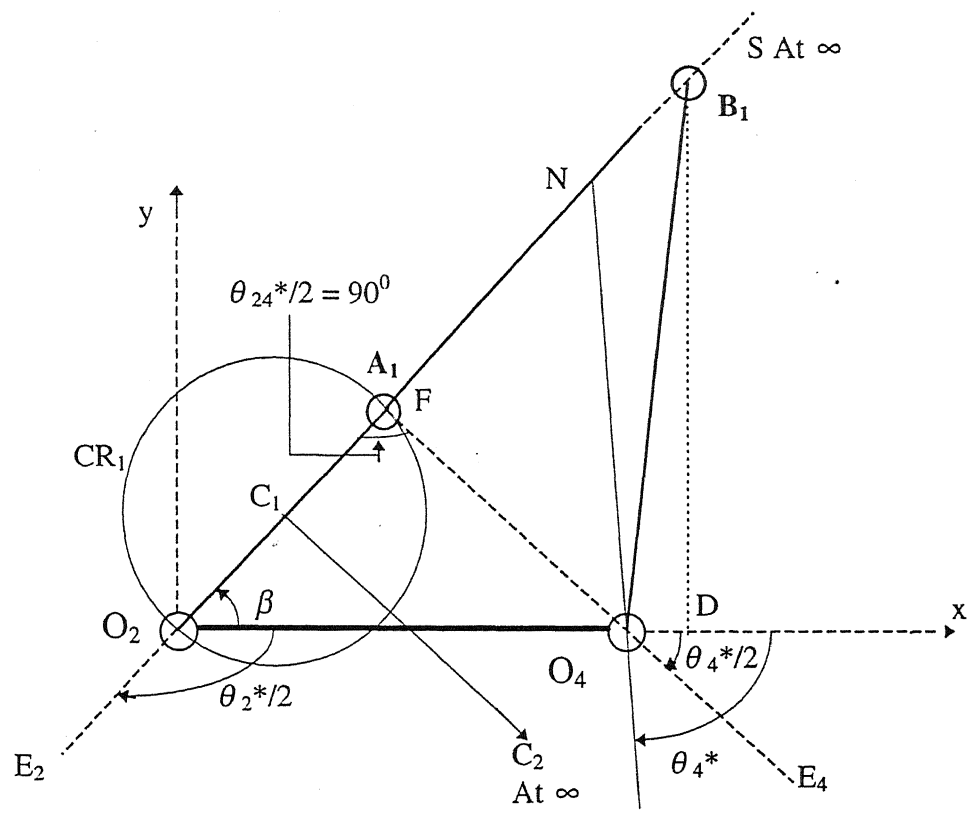
[Fig 2.4]



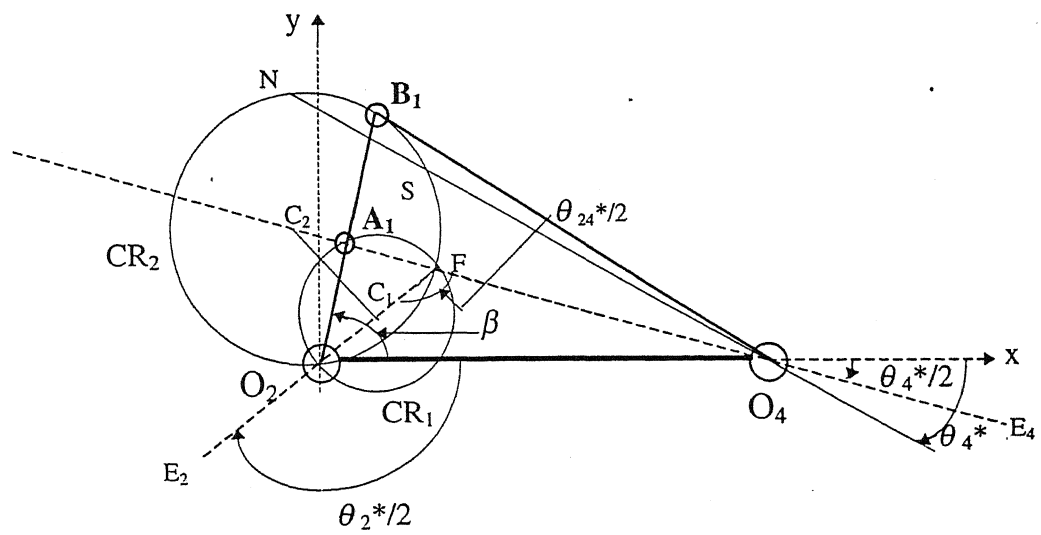
[Fig 2.5]



[Fig 2.6]

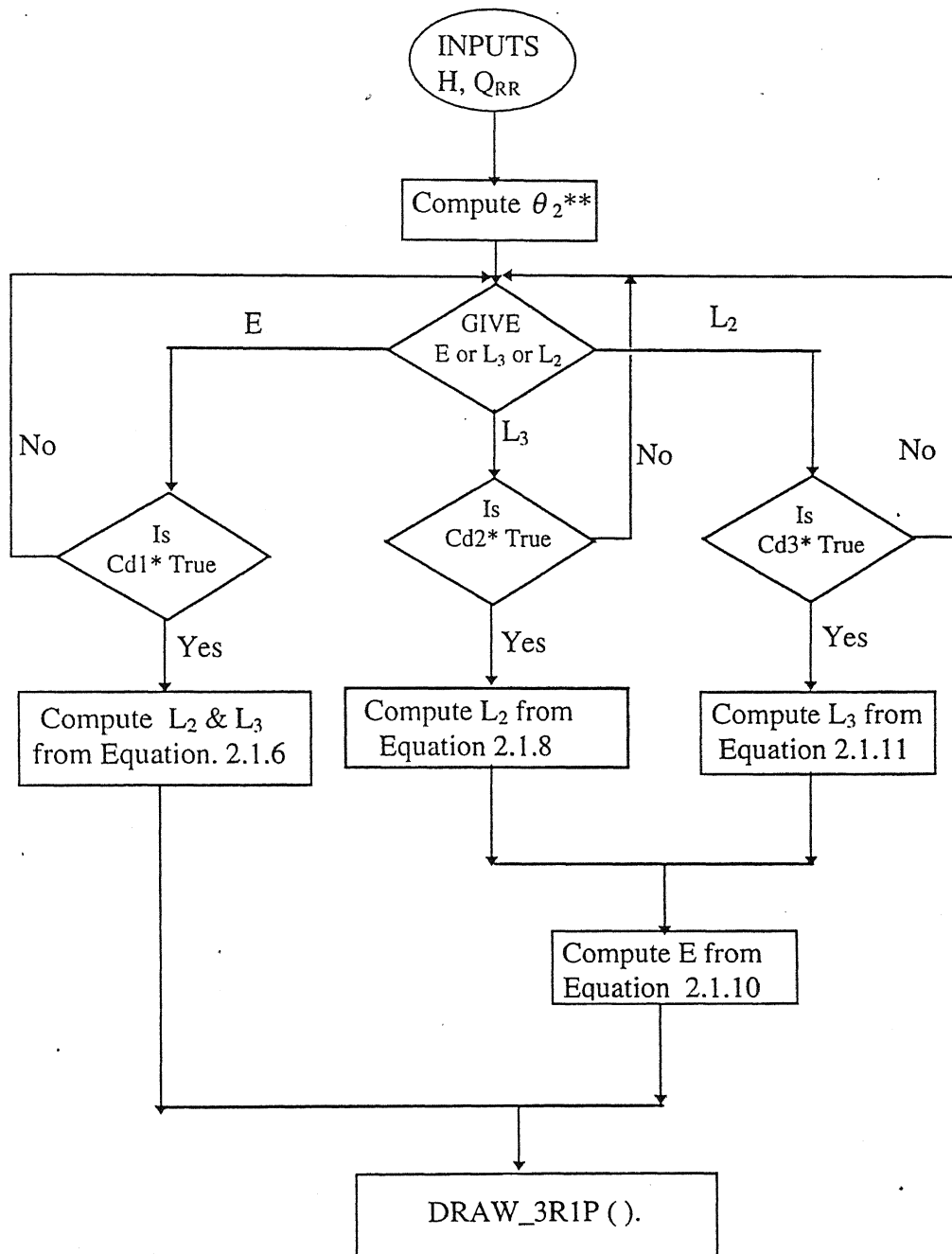


[Fig 2.7]



[Fig 2.8]

[DEAD-CENTER PROBLEMS FOR 3R-1P MECHANISMS]



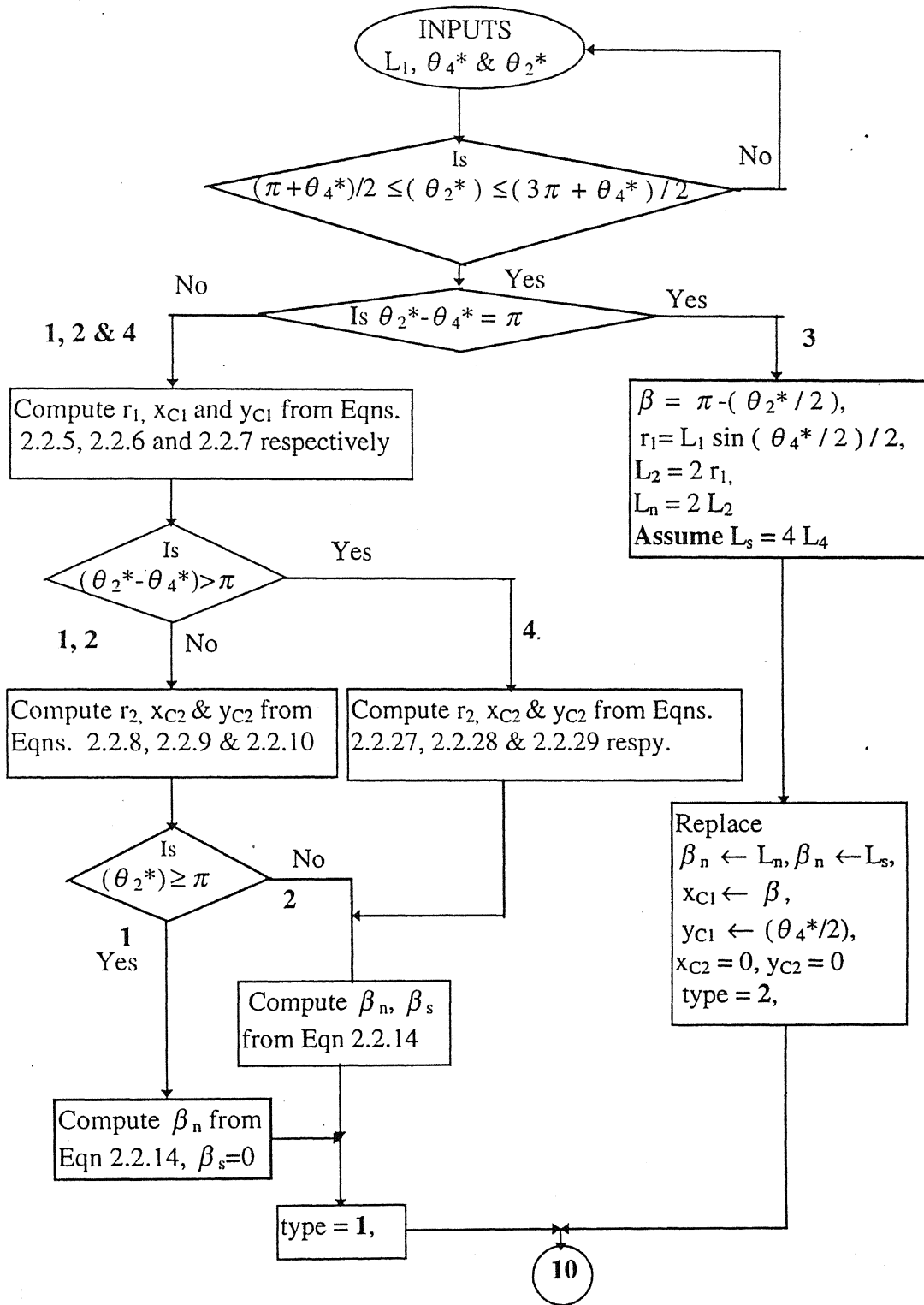
$$* Cd1 \rightarrow [|\cos(\theta_2^{**}) - (2E/H)\sin(\theta_2^{**})| \leq 1]$$

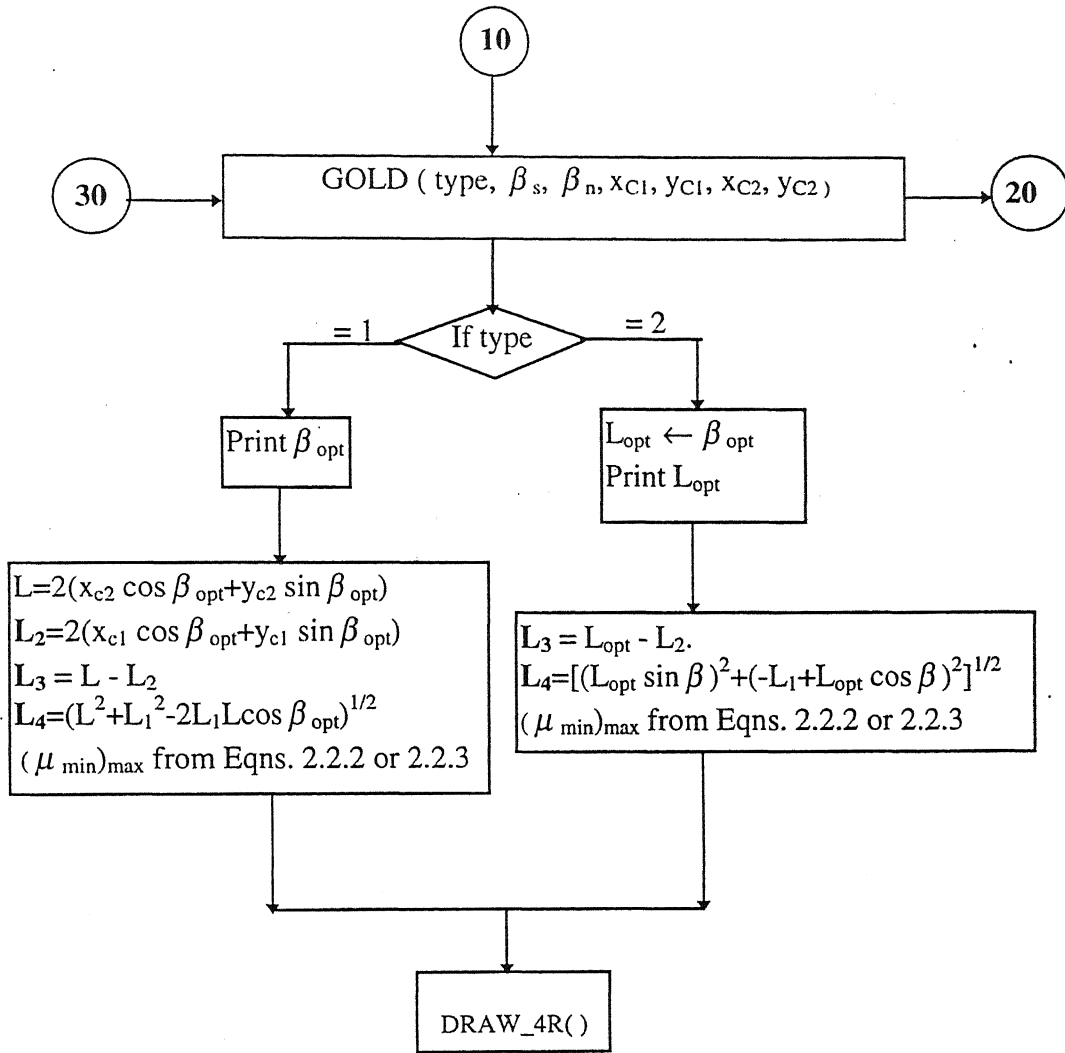
$$* Cd2 \rightarrow [(H^2/2) \geq L_3^2 (1 - \cos(\theta_2^{**}))]$$

$$* Cd3 \rightarrow [(H^2/2) \geq L_2^2 (1 + \cos(\theta_2^{**}))]$$

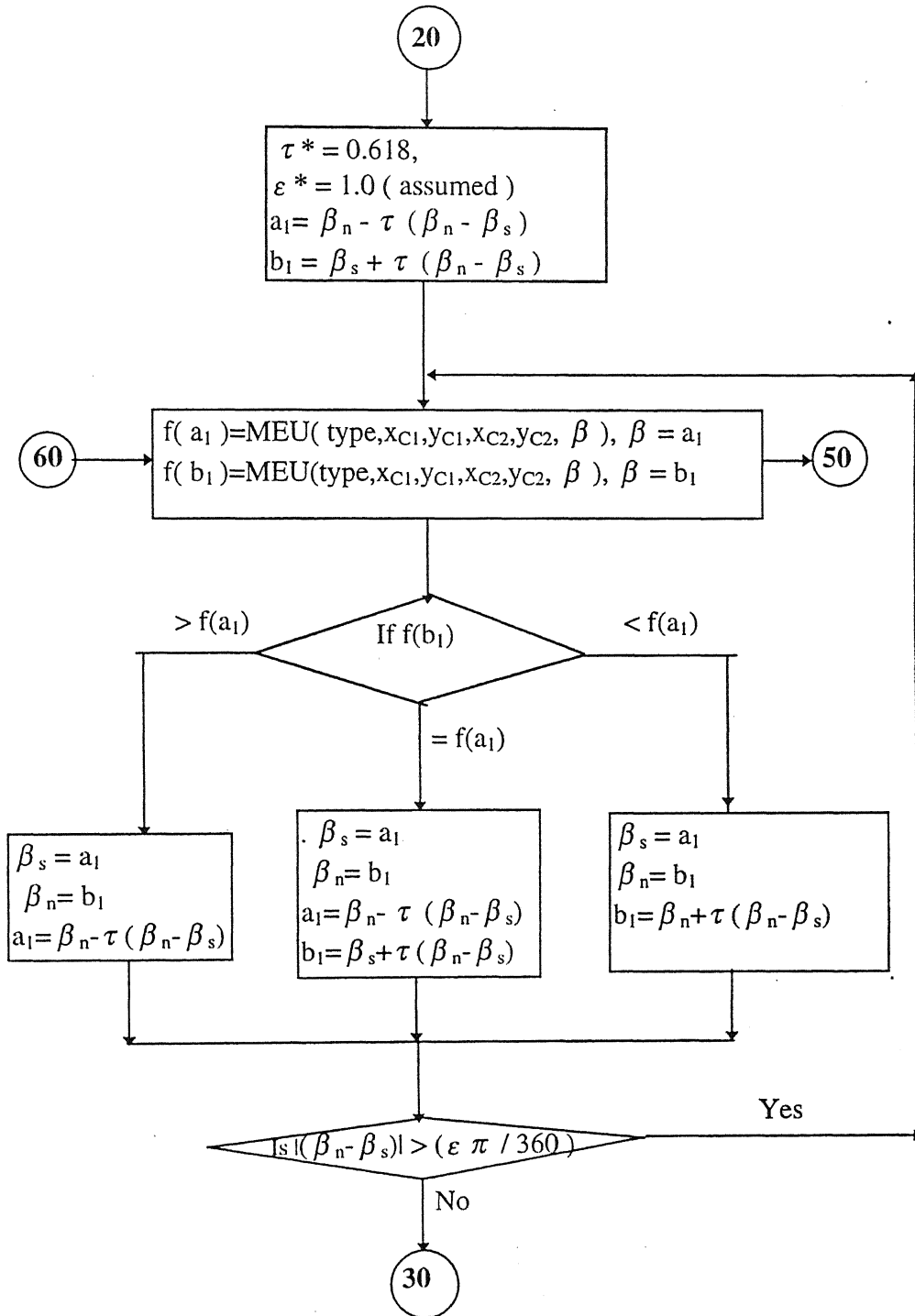
[Chart 2.1]

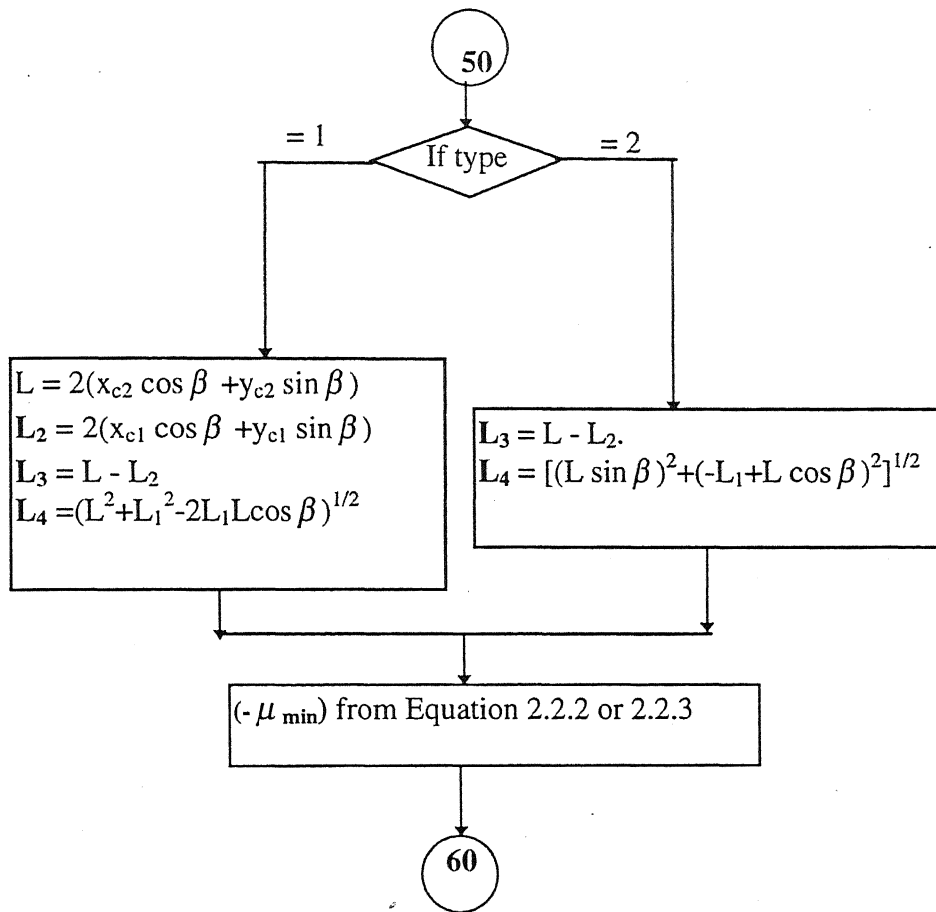
[DEAD-CENTER PROBLEMS FOR CRANK-ROCKER MECHANISMS]



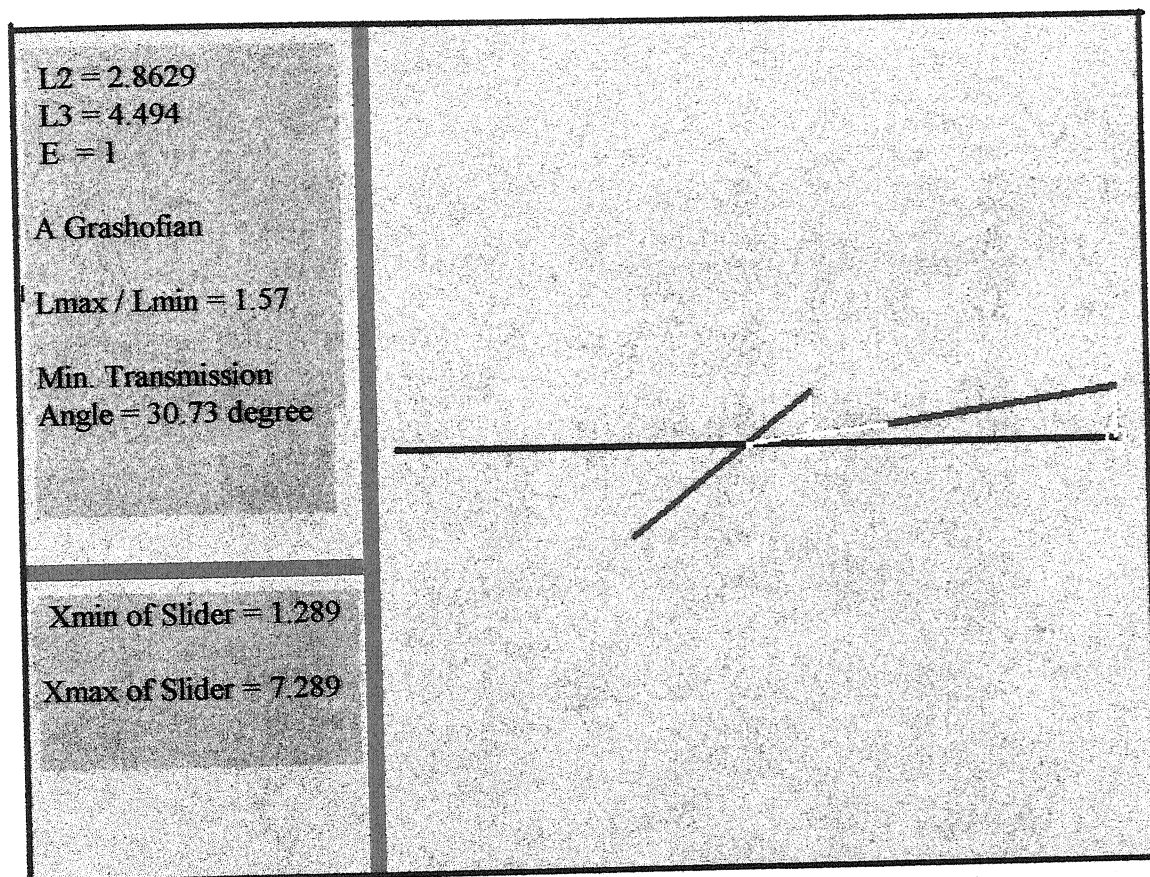


[GOLDEN SECTION SEARCH METHOD]

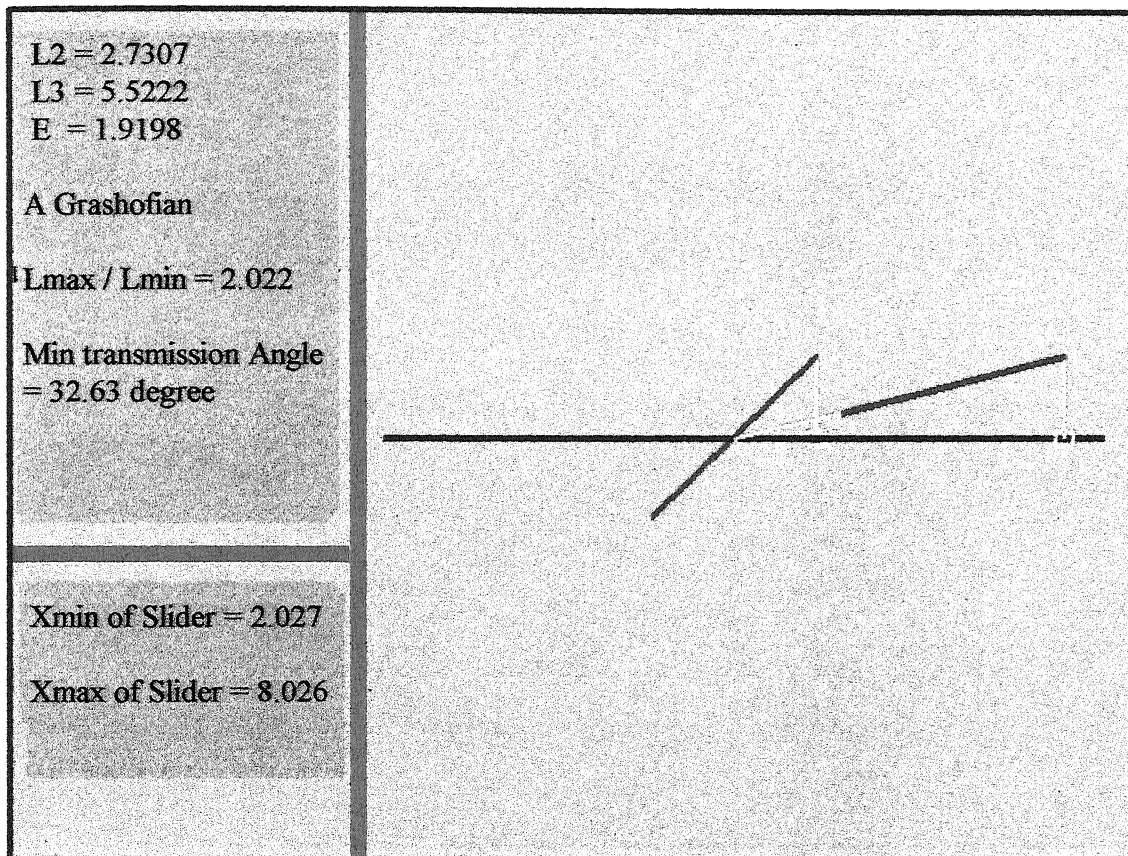




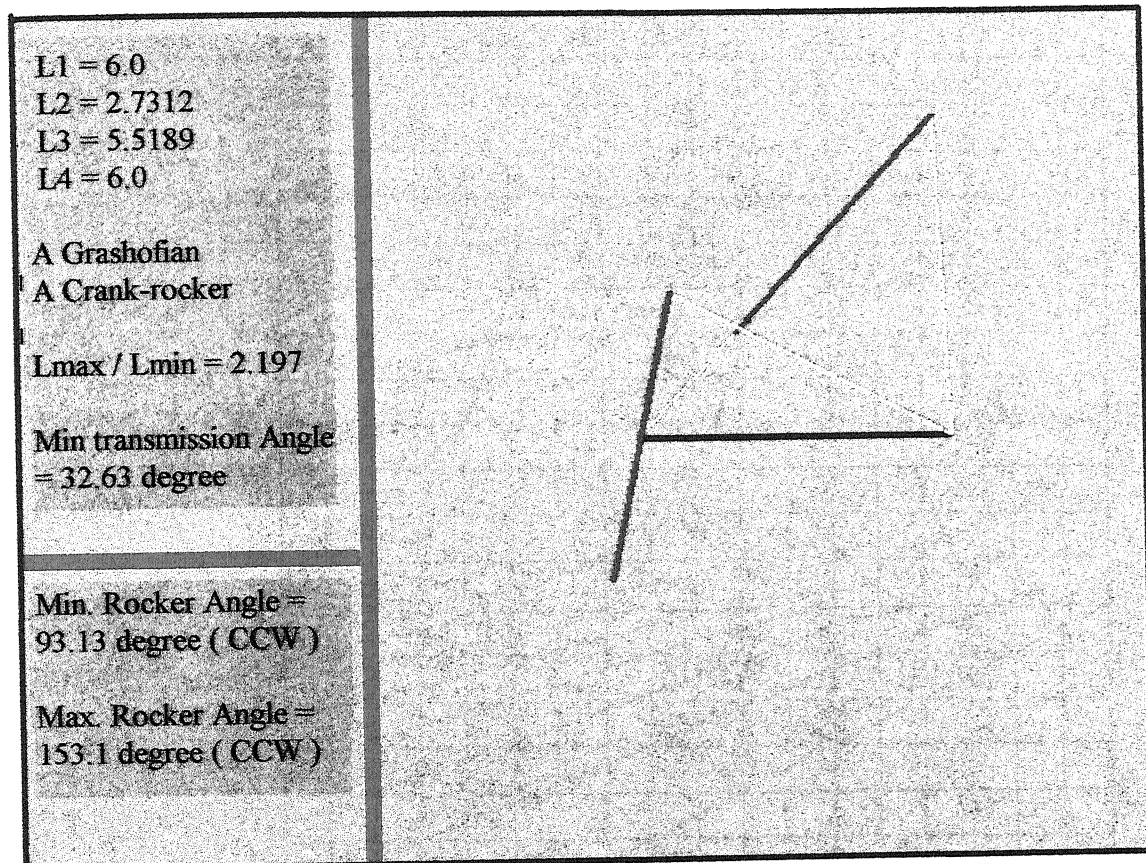
[Chart 2.2]



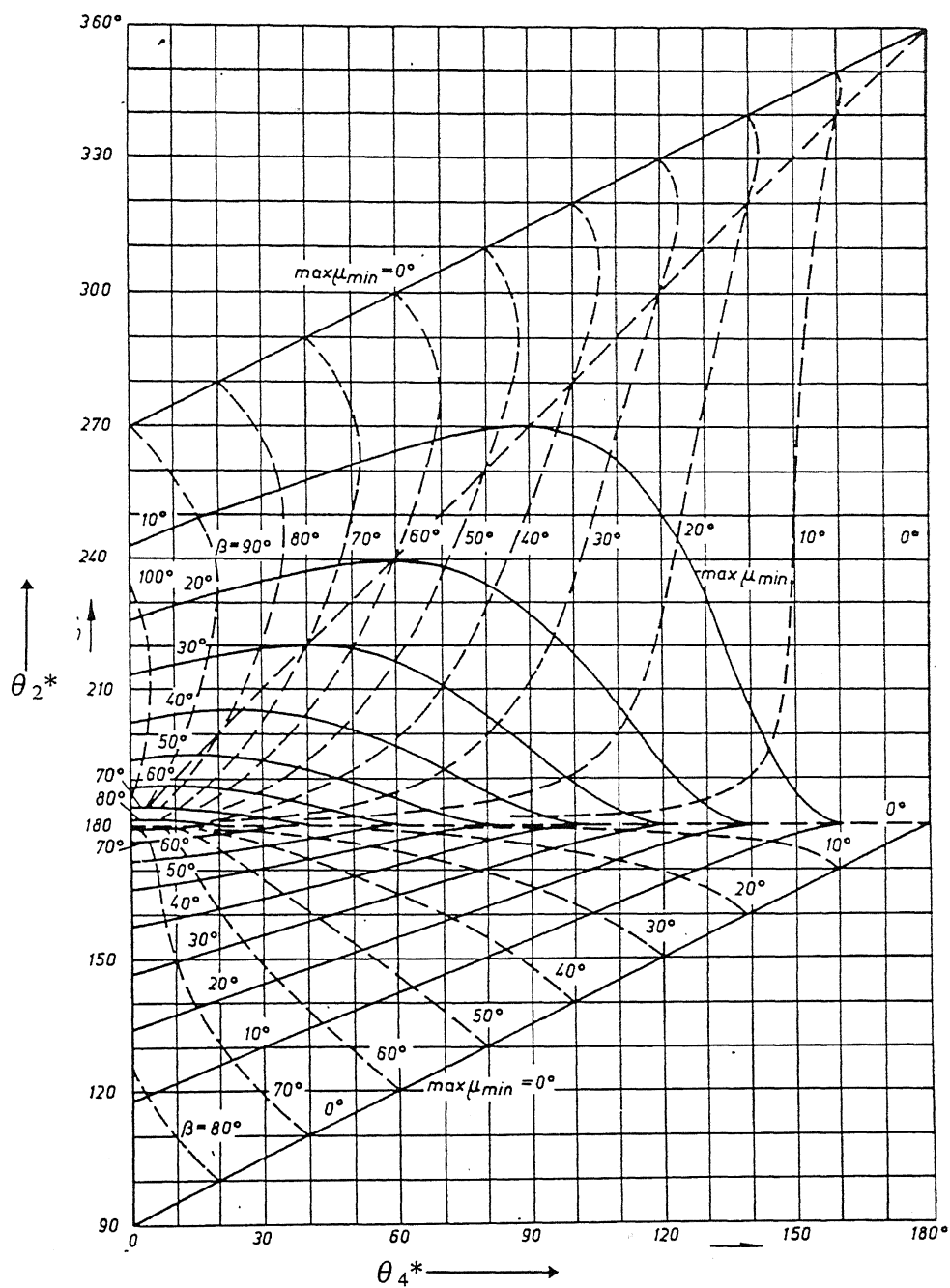
[Plot 2.1]



[Plot 2.2]



[Plot 2.3]

Volmer's Nomogram^[10]

[Plot 2.4]

3. APPROACH FOR PATH AND FUNCTION GENERATION

In this chapter we shall discuss a *least square technique* for synthesizing 4R mechanisms for path generation and function generation (or coordinated input-output) problems. This approach is generally used for more than three design positions.

3.1 . PATH GENERATION

Here the coordinates of some points on the specified path are available. These points may or may not be coordinated with the input angles. If the coordinated input angles are not specified then one can assume these angles which are required in this method. Now we have to synthesize a 4R mechanism such that its coupler point approximately passes through the specified points.

Figure 3.1 shows a 4R mechanism, O_2ABCO_4 with unknown design parameters $R_1, R_2, R_3, R_4, R_5, R_6, \hat{\alpha}, \hat{\beta}, \phi$ and ψ out of which α and β are assumed initially. Suppose there are n points through which the coupler point C has to pass and the coupler point at the i^{th} position is specified by its polar coordinates (r_i, δ_i) with the coordinated input represented by θ_{2i} .

Now, writing the loop-closure equations for the loop OO_2AC_1O , we get .

$$R_1 \cos \hat{\alpha} + R_2 \cos \theta_{2i} + R_3 \cos \psi_i = r_i \cos \delta_i \quad [3.1.1]$$

and

$$R_1 \sin \hat{\alpha} + R_2 \sin \theta_{2i} + R_3 \sin \psi_i = r_i \sin \delta_i . \quad [3.1.2]$$

Similarly, for the loop OO_4BC_1O , we have

$$R_4 \cos \hat{\beta} + R_5 \cos \theta_{4i} + R_6 \cos \phi_i = r_i \cos \delta_i \quad [3.1.3]$$

and

$$R_4 \sin \hat{\beta} + R_5 \sin \theta_{4i} + R_6 \sin \phi_i = r_i \sin \delta_i. \quad [3.1.4]$$

Eliminating ψ_i from Equations 3.1.1 and 3.1.2, we get

$$(R_1 \cos \hat{\alpha} + R_2 \cos \theta_{2i} - r_i \cos \delta_i)^2 + (R_1 \sin \hat{\alpha} + R_2 \sin \theta_{2i} - r_i \sin \delta_i)^2 = R_3^2$$

or

$$\begin{aligned} r_i^2 + R_2^2 + R_1^2 + 2 R_1 R_2 \cos (\theta_{2i} - \hat{\alpha}) - 2 R_1 r_i \cos (\hat{\alpha} - \delta_i) \\ - 2 R_2 r_i \cos (\theta_{2i} - \delta_i) = R_3^2 \end{aligned}$$

or

$$\begin{aligned} R_1 [2 r_i \cos (\hat{\alpha} - \delta_i)] + R_2 [2 r_i \cos (\theta_{2i} - \delta_i)] + (R_3^2 - R_2^2 - R_1^2) \\ = r_i^2 + R_1 R_2 [2 \cos (\theta_{2i} - \hat{\alpha})]. \quad [3.1.5] \end{aligned}$$

Similarly, eliminating θ_{4i} from Equations 3.1.3 and 3.1.4, we get

$$\begin{aligned} R_4 [2 r_i \cos (\hat{\beta} - \delta_i)] + R_6 [2 r_i \cos (\phi_i - \delta_i)] + (R_5^2 - R_4^2 - R_6^2) \\ = r_i^2 + R_4 R_6 [2 \cos (\phi_i - \hat{\beta})]. \quad [3.1.6] \end{aligned}$$

Substituting

$$\left. \begin{aligned} D_1 &= R_1, & D_5 &= R_4, \\ D_2 &= R_2, & D_6 &= R_6, \\ D_3 &= R_3^2 - R_2^2 - R_1^2, & D_7 &= R_5^2 - R_4^2 - R_6^2 \\ D_4 &= R_1 R_2, & \text{and } D_8 &= R_4 R_6 \end{aligned} \right| \quad [3.1.7]$$

Equations 3.1.5 and 3.1.6 are linearised in the design parameters D_i 's ($i = 1$ to 8) as

$$\begin{aligned} D_1 [2 r_i \cos (\hat{\alpha} - \delta_i)] + D_2 [2 r_i \cos (\theta_{2i} - \delta_i)] + D_3 \\ - r_i^2 - D_4 [2 \cos (\theta_{2i} - \hat{\alpha})] = 0 \quad [3.1.8] \end{aligned}$$

and

$$D_5 [2 r_i \cos (\hat{\beta} - \delta_i)] + D_6 [2 r_i \cos (\phi_i - \delta_i)] + D_7 \\ - r_i^2 - D_8 [2 \cos (\phi_i - \hat{\beta})] = 0. \quad [3.1.9]$$

The design parameters are nothing but functions of the link lengths. If the coupler point passes exactly through the prescribed points at the desired input angles then Equations 3.1.8 and 3.1.9 are perfectly satisfied. Here, only r_i , δ_i and θ_{2i} ($i = 1$ to n) are available, but while solving Equations 3.1.8 and 3.1.9, it requires some assumptions

(say $\hat{\alpha}$, $\hat{\beta}$ and ϕ_i). At this stage, it must be appreciated that Equations 3.1.8 and 3.1.9 are not satisfied exactly. Now let e_{1i} and e_{2i} represent the error in the loop-closure equations 3.1.8 and 3.1.9, respectively, (at the i^{th} design position). Thus,

$$e_{1i} = D_1 [2 r_i \cos (\hat{\alpha} - \delta_i)] + D_2 [2 r_i \cos (\theta_{2i} - \delta_i)] + D_3 \\ - r_i^2 - D_4 [2 \cos (\theta_{2i} - \hat{\alpha})] \quad [3.1.10]$$

and

$$e_{2i} = D_5 [2 r_i \cos (\hat{\beta} - \delta_i)] + D_6 [2 r_i \cos (\phi_i - \delta_i)] + D_7 \\ - r_i^2 - D_8 [2 \cos (\phi_i - \hat{\beta})]. \quad [3.1.11]$$

It is obvious from Equation 3.1.7 that

$$D_4 = D_1 D_2 \quad \text{and} \quad D_8 = D_5 D_6.$$

Thus D_4 and D_8 make the Equations 3.1.10 and 3.1.11 non-linear. Let us first solve Equation 3.1.10 as explained below.

Let

$$D_4 = D_1 D_2 = \lambda \quad [3.1.12]$$

$$\text{and} \quad D_p = l_p + \lambda m_p, \text{ for } p = 1, 2, 3. \quad [3.1.13]$$

From Equations 3.1.12 and 3.1.13

$$\lambda = (l_1 + \lambda m_1)(l_2 + \lambda m_2). \quad [3.1.14]$$

Therefore, from equation 3.1.10

$$e_{li} = e_{li,l} + \lambda e_{li,m}, \quad [3.1.15]$$

where

$$e_{li,l} = l_1 [2 r_i \cos(\hat{\alpha} - \delta_i)] + l_2 [2 r_i \cos(\theta_{2i} - \delta_i)] + l_3 - r_i^2$$

and

$$e_{li,m} = m_1 [2 r_i \cos(\hat{\alpha} - \delta_i)] + m_2 [2 r_i \cos(\theta_{2i} - \delta_i)] + m_3 - 2 \cos(\theta_{2i} - \hat{\alpha}).$$

In the ideal case, $e_{li} = 0$. To achieve this we would like to get l 's and m 's such that $e_{li,l} = 0$ and $e_{li,m} = 0$.

Similarly, for solving Equation 3.1.11,

let

$$D_8 = D_5 D_6 = \eta \quad [3.1.16]$$

and

$$D_p = u_p + \eta v_p, \text{ for } p = 5, 6, 7. \quad [3.1.17]$$

From Equations 3.1.16 and 3.1.17, we get

$$\eta = (u_1 + \eta v_1)(u_2 + \eta v_2). \quad [3.1.18]$$

Therefore, from equation 3.1.11

$$e_{2i} = e_{2i,u} + \lambda e_{2i,v}, \quad [3.1.19]$$

where

$$e_{2i,u} = u_1 [2 r_i \cos(\hat{\beta} - \delta_i)] + u_2 [2 r_i \cos(\phi_i - \delta_i)] + u_3 - r_i^2$$

and

$$e_{2i,v} = v_1 [2 r_i \cos (\hat{\beta} - \delta_i)] + v_2 [2 r_i \cos (\phi_i - \delta_i)] + v_3 - 2 \cos (\phi_i - \hat{\beta}).$$

Again in the ideal case $e_{2i} = 0$. To achieve this we would like to get u 's and v 's such that $e_{2i,u} = 0$ and $e_{2i,v} = 0$.

Using the least square technique, now we shall minimize the overall error (e), which is expressed as

$$e = \sum_{i=1}^n w_{t_i} (e_{1i,l}^2 + e_{1i,m}^2 + e_{2i,u}^2 + e_{2i,v}^2), \quad [3.1.20]$$

where the *error functions* are weighted with w_{t_i} for the i^{th} position.

It helps the designer to minimize the error more at the positions where he / she needs to do so. In other words, after some number of iterations, the designer can obtain a better solution with the help of these weighting functions w_{t_i} . In the first iteration, w_{t_i} are taken as unity for all i 's and then it is checked at which design positions errors are more than acceptable. Thereafter, the w_{t_i} 's are modified accordingly. After the first iteration, we have taken w_{t_i} equal to the structural error at the i^{th} position. By iteration with these weighting functions the user can minimize the structural error at some critical points.

Now, for the minimum value of e , we set the first partial derivatives to zero. Thus, we get

$$\frac{\partial}{\partial l_1} \left[\sum_{i=1}^n w_{t_i} e_{1i,l}^2 \right] = \frac{\partial}{\partial l_2} \left[\sum_{i=1}^n w_{t_i} e_{1i,l}^2 \right] = \frac{\partial}{\partial l_3} \left[\sum_{i=1}^n w_{t_i} e_{1i,l}^2 \right] = 0, \quad [3.1.21a]$$

$$\frac{\partial}{\partial m_1} \left[\sum_{i=1}^n w_{t_i} e_{1i,m}^2 \right] = \frac{\partial}{\partial m_2} \left[\sum_{i=1}^n w_{t_i} e_{1i,m}^2 \right] = \frac{\partial}{\partial m_3} \left[\sum_{i=1}^n w_{t_i} e_{1i,m}^2 \right] = 0, \quad [3.1.21b]$$

$$\frac{\partial}{\partial u_1} \left[\sum_{i=1}^n w_{ti} e_{2i,u}^2 \right] = \frac{\partial}{\partial u_2} \left[\sum_{i=1}^n w_{ti} e_{2i,u}^2 \right] = \frac{\partial}{\partial u_3} \left[\sum_{i=1}^n w_{ti} e_{2i,u}^2 \right] = 0$$

[3.1.21c]

and

$$\frac{\partial}{\partial v_1} \left[\sum_{i=1}^n w_{ti} e_{2i,v}^2 \right] = \frac{\partial}{\partial v_2} \left[\sum_{i=1}^n w_{ti} e_{2i,v}^2 \right] = \frac{\partial}{\partial v_3} \left[\sum_{i=1}^n w_{ti} e_{2i,v}^2 \right] = 0.$$

[3.1.21d]

Now, onwards the summation indices i and n are omitted.

Equation 3.1.21a gives

$$\begin{aligned} & l_1 \sum [w_{ti} \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}^2] \\ & + l_2 \sum [w_{ti} \{ 2 r_i \cos (\theta_{2i} - \delta_i) \} \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}] \\ & + l_3 \sum [w_{ti} \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}] \\ & = \sum [w_{ti} r_i^2 \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}], \end{aligned}$$

[3.1.22a]

$$\begin{aligned} & l_1 \sum [w_{ti} \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \} \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}] \\ & + l_2 \sum [w_{ti} \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}^2] \\ & + l_3 \sum [w_{ti} \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}] \\ & = \sum [w_{ti} r_i^2 \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}] \end{aligned}$$

[3.1.22b]

and

$$\begin{aligned} & l_1 \sum [w_{ti} \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}] \\ & + l_2 \sum [w_{ti} \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}] \\ & + l_3 \sum [w_{ti}] \\ & = \sum [w_{ti} r_i^2]. \end{aligned}$$

[3.1.22c]

Thus, these three equations can be solved by using *Cramer's rule* to determine unknowns l_1 , l_2 and l_3 .

In a similar manner, Equation 3.1.21b gives

$$\begin{aligned}
 m_1 \sum [wt_i \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}^2] \\
 + m_2 \sum [wt_i \{ 2 r_i \cos (\theta_{2i} - \delta_i) \} \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}] \\
 + m_3 \sum [wt_i \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}] \\
 = \sum [wt_i \{ 2 \cos (\theta_{2i} - \hat{\alpha}) \} \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}], \\
 \end{aligned} \tag{3.1.23a}$$

$$\begin{aligned}
 m_1 \sum [wt_i \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \} \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}] \\
 + m_2 \sum [wt_i \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}^2] \\
 + m_3 \sum [wt_i \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}] \\
 = \sum [wt_i \{ 2 \cos (\theta_{2i} - \hat{\alpha}) \} \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}] \\
 \end{aligned} \tag{3.1.23b}$$

and

$$\begin{aligned}
 m_1 \sum [wt_i \{ 2 r_i \cos (\hat{\alpha} - \delta_i) \}] \\
 + m_2 \sum [wt_i \{ 2 r_i \cos (\theta_{2i} - \delta_i) \}] \\
 + m_3 \sum [wt_i] \\
 = \sum [wt_i \{ 2 \cos (\theta_{2i} - \hat{\alpha}) \}]. \\
 \end{aligned} \tag{3.1.23c}$$

Again these three equations can be solved by using *Cramer's rule* to determine m_1 , m_2 and m_3 . Once l_1 , l_2 , l_3 and m_1 , m_2 , m_3 are known, we can determine λ (from Equation 3.1.14), and then D_1 , D_2 and D_3 (from Equation 3.1.13) and hence R_1 , R_2 and R_3 (from Equation 3.1.7).

Now we can determine ψ_i 's, $i=1$ to n , by using Equation 3.1.1 or 3.1.2.

Assuming a suitable value for ϕ_1 , we can determine ϕ_i (for $i = 2$ to n) by using the following relations.

$$\phi_i = \phi_1 + (\phi_i - \psi_i), (\text{for } i = 2 \text{ to } n) \quad [3.1.24]$$

Thus, we have two values of λ (say λ_1 and λ_2) and correspondingly two values of R_1, R_2 and R_3 (say $R_1[1], R_2[1], R_3[1]$ and $R_1[2], R_2[2], R_3[2]$) and hence two values of ϕ_i 's (for $i = 2$ to n).

In the same manner, Equations 3.1.21c and 3.1.21d give

$$\begin{aligned} u_1 \sum [wt_i \{ 2 r_i \cos(\hat{\beta} - \delta_i) \}^2] \\ + u_2 \sum [wt_i \{ 2 r_i \cos(\phi_i - \delta_i) \} \{ 2 r_i \cos(\hat{\beta} - \delta_i) \}] \\ + u_3 \sum [wt_i \{ 2 r_i \cos(\hat{\beta} - \delta_i) \}] \\ = \sum [wt_i r_i^2 \{ 2 r_i \cos(\hat{\beta} - \delta_i) \}], \end{aligned} \quad [3.1.25a]$$

$$\begin{aligned} u_1 \sum [wt_i \{ 2 r_i \cos(\hat{\beta} - \delta_i) \} \{ 2 r_i \cos(\phi_i - \delta_i) \}] \\ + u_2 \sum [wt_i \{ 2 r_i \cos(\phi_i - \delta_i) \}^2] \\ + u_3 \sum [wt_i \{ 2 r_i \cos(\phi_i - \delta_i) \}] \\ = \sum [wt_i r_i^2 \{ 2 r_i \cos(\phi_i - \delta_i) \}], \end{aligned} \quad [3.1.25b]$$

$$\begin{aligned} u_1 \sum [wt_i \{ 2 r_i \cos(\hat{\beta} - \delta_i) \}] \\ + u_2 \sum [wt_i \{ 2 r_i \cos(\phi_i - \delta_i) \}] \\ + u_3 \sum [wt_i] \\ = \sum [wt_i r_i^2], \end{aligned} \quad [3.1.25c]$$

$$\begin{aligned}
& v_1 \sum [wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \}^2] \\
& + v_2 \sum [wt_i \{ 2 r_i \cos (\phi_i - \delta_i) \} \{ 2 r_i \cos (\hat{\beta} - \delta_i) \}] \\
& + v_3 \sum [wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \}] \\
& = \sum [wt_i \{ 2 r_i \cos (\phi_i - \hat{\beta}) \} \{ 2 r_i \cos (\hat{\beta} - \delta_i) \}], \\
& \quad \quad \quad [3.1.26a]
\end{aligned}$$

$$\begin{aligned}
& v_1 \sum [wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \} \{ 2 r_i \cos (\phi_i - \delta_i) \}] \\
& + v_2 \sum [wt_i \{ 2 r_i \cos (\phi_i - \delta_i) \}^2] \\
& + v_3 \sum [wt_i \{ 2 r_i \cos (\phi_i - \delta_i) \}] \\
& = \sum [wt_i \{ 2 r_i \cos (\phi_i - \hat{\beta}) \} \{ 2 r_i \cos (\phi_i - \delta_i) \}] \\
& \quad \quad \quad [3.1.26b]
\end{aligned}$$

and

$$\begin{aligned}
& v_1 \sum [wt_i \{ 2 r_i \cos (\hat{\beta} - \delta_i) \}] \\
& + v_2 \sum [wt_i \{ 2 r_i \cos (\phi_i - \delta_i) \}] \\
& + v_3 \sum [wt_i] \\
& = \sum [wt_i \{ 2 r_i \cos (\phi_i - \hat{\beta}) \}]. \\
& \quad \quad \quad [3.1.26c]
\end{aligned}$$

Again, from Equations 3.1.25a, 3.1.25b and 3.1.25c we determine u_1 , u_2 and u_3 . Similarly v_1 , v_2 and v_3 are obtained from Equations 3.1.26a, 3.1.26b and 3.1.26c. Therefore we get η (from Equation 3.1.18), and then D_4 , D_5 and D_6 (from Equation 3.1.17) and hence R_4 , R_5 and R_6 (from Equation 3.1.7). There are two values of ϕ_i 's (for $i = 2$ to n) and each value of ϕ_i 's (for $i = 2$ to n) gives two values η [say the first value of ϕ_i 's (for $i = 2$ to n) gives η_{11} , η_{12} and the second value of ϕ_i 's (for $i = 2$ to n) gives η_{21} , η_{22}]. Hence there are four values η and therefore, we get four values of R_4 , R_5 and R_6 , which are denoted as

$$R_4[1][1], R_4[1][2], R_4[2][1], R_4[2][2],$$

$$R_5[1][1], R_5[1][2], R_5[2][1], R_5[2][2]$$

and

$$R_6[1][1], R_6[1][2], R_6[2][1], R_6[2][2].$$

Hence, we have four solutions as

$$[i] R_1[1], R_2[1], R_3[1], R_4[1][1], R_5[1][1], R_6[1][1] \hat{\alpha}, \hat{\beta}, \phi_1,$$

$$[ii] R_1[1], R_2[1], R_3[1], R_4[1][2], R_5[1][2], R_6[1][2] \hat{\alpha}, \hat{\beta}, \phi_1,$$

$$[iii] R_1[2], R_2[2], R_3[2], R_4[2][1], R_5[2][1], R_6[2][1] \hat{\alpha}, \hat{\beta}, \phi_1$$

and

$$[iv] R_1[2], R_2[2], R_3[2], R_4[2][2], R_5[2][2], R_6[2][2] \hat{\alpha}, \hat{\beta}, \phi_1.$$

Now, the parameters ($R_1, R_2, R_3, R_4, R_5, R_6, \hat{\alpha}, \hat{\beta}$ and ϕ_1) can be converted into the link lengths (L_1, L_2, L_3 and L_4) and the polar coordinates (R, γ) of the coupler point, such that

$$\begin{aligned} L_1 &= O_2O_4 = [(R_4 \cos \hat{\beta} - R_1 \cos \hat{\alpha})^2 + (R_4 \sin \hat{\beta} - R_1 \sin \hat{\alpha})^2]^{1/2}, \\ L_2 &= O_2A = R_2, \\ L_3 &= AB = [R_6^2 + R_3^2 - 2 R_6 R_3 \cos (\phi_1 - \psi_1)]^{1/2}, \\ L_4 &= O_4B = R_5, \\ R &= AC = R_3, \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right| [3.1.27]$$

$$\text{and } \gamma = \angle CAB = \sin^{-1} [R_6 \sin (\phi_1 - \psi_1) / L_3].$$

These new parameters (L_1, L_2, L_3, L_4, R and γ) are used in the animation of a 4R mechanism for path generation.

Hence, we can find $L_1[k]$, $L_2[k]$, $L_3[k]$, $L_4[k]$, $R[k]$ and $\gamma[k]$ for each k^{th} solution. It is not guaranteed that all the above solutions give acceptable results. It mainly depends upon the assumed values of $\hat{\alpha}$, $\hat{\beta}$ and ϕ_1 needed to solve the equations. By changing these values one can arrive at better solutions.

Flow Chart - Chart 3.1 shows the steps used in the programming. Here are some mathematical bridges (Quad1, Sqrt1, Quad2, Sqrt2, see in Chart 3.1), without getting these positive one can not move further. These expressions are controlled by changing the assumed values of $\hat{\alpha}$, $\hat{\beta}$ and ϕ_1 . Some bridges are being controlled automatically and others need manual controlling. Regarding the selection of $\hat{\alpha}$ and $\hat{\beta}$, one should choose the parameters such that most of the specified points lie near the perpendicular to the line O_2O_4 and to get a crank-rocker or a double-rocker mechanism, the specified points must lie on one side of the line O_2O_4 .

The present software can also handle some constraints like

- [a] the ratio (L_{\max} / L_{\min}), where L_{\max} and L_{\min} are the maximum and minimum values of link lengths of the mechanism,
- [b] the designed mechanism must be a crank-rocker and
- [c] the structural error at a specified point must be zero i.e, the coupler point must pass through a specific point on the path.

The ratio L_{\max} / L_{\min} is controlled by $\hat{\beta}$ whereas a crank-rocker can be ensured by ϕ_1 . Here, to minimize the structural error, the least square error function is weighted by the distance between the specified and generated points at each prescribed θ_2 . Obviously some trials are necessary to arrive at an acceptable solution. Here some *steps* are given, following which one can easily run the program quickly to get a good result.

- [i] Give n , (x_i, y_i) or (r_i, δ_i) and θ_{2i} .

[ii] Select any one of the solutions (1, 2, 3 and 4) and assume ($\hat{\alpha}$, $\hat{\beta}$ and ϕ_1) as default values. Initially the constraints are left out.

[iii] Quad1 is automatically controlled, but controlling of Sqrt1 may require a manual change of $\hat{\alpha}$. Similarly, Quad1 is controlled automatically but Sqrt2 is controlled by changing $\hat{\beta}$ manually. The program itself takes care of the constraints on the ratio (L_{\max} / L_{\min}) and the crank-rocker compulsion.

[iv] Check the other three solutions and choose the best one. Now, if the structural errors are large, then go to step [ii] and change the assumptions keeping in mind the previous results and the assumptions made. After two to three trials, one can guess the suitable values of $\hat{\alpha}$, $\hat{\beta}$ and ϕ_1 . The variation of ϕ_1 can help to reach a good solution.

[v] For better results, choose an option of weighting the error function. Weighting on the error functions are possible only in the case of crank-rocker and double-rocker mechanisms. One should check the other three solutions if one solution exists.

If instead of prescribed points, a path is given, then start solving the problem by considering 6 points on the path at equal steps of θ_2 . Then increase the number of points to 8, 10 and 12. The present software considers a maximum of twelve points but one can replace it by any number. However, care should be taken regarding the memory and the speed of the computing system.

An Illustration - Let us consider a path generated by a crank-rocker mechanism (shown in Plot 3.1) with

$$\begin{aligned} L_1 &= 2 \text{ unit,} \\ L_2 &= 1 \text{ unit,} \\ L_3 &= 2 \text{ unit,} \\ L_4 &= 2 \text{ unit,} \\ R &= 3 \text{ unit} \\ \text{and } \gamma &= 120^\circ. \end{aligned}$$

Number of points selected = 12, which are

Point	x coordinate	y coordinate	θ_2
1	-1.8906	-0.8028	0.0
2	-2.0701	1.1157	30.0
3	-2.2042	2.1651	60.0
4	-2.5835	2.525	90.0
5	-3.0845	2.3894	120.0
6	-3.5495	1.8414	150.0
7	-3.8435	0.9564	180.0
8	-3.8451	-0.1465	210.0
9	-3.4730	-1.2682	240.0
10	-2.7701	-2.1518	270.0
11	-1.9771	-2.5584	300.0
12	-1.5689	-2.2525	330.0

For this problem, assuming $\hat{\alpha}$, $\hat{\beta}$, ϕ_1 and L_{\max} / L_{\min} as default values ($\hat{\alpha} = 56^\circ$, $\hat{\beta} = 354^\circ$, $\phi_1 = 166^\circ$ and $L_{\max} / L_{\min} = 20$) we get the following results.

Solution no.	$\hat{\alpha}$	$\hat{\beta}$	ϕ_1	L_{\max} / L_{\min}	Type of mechanism	(NP)Number of specified points through which the coupler point approximately passes.
1	56	354	166	19.62	Crank - rocker	2
2	56	354	166	1.497	Non Grashofian	6
3	56	354	166	10.13	Non Grashofian	2
4	56	354	166	2.289	Non Grashofian	8

In the next iteration, an attempt is made to get a crank-rocker mechanism for solution no. 4. The final results turn out as $L_{\max} / L_{\min} = 1.968$ and $NP = 4$ for a crank-rocker mechanism. Then for a new set of assumptions ($\hat{\alpha} = 50^\circ$, $\hat{\beta} = 50^\circ$, $\phi_1 = 90^\circ$ and $L_{\max} / L_{\min} = 20$), the following results are obtained.

Solution no.	$\hat{\alpha}$	$\hat{\beta}$	ϕ_1	L_{\max} / L_{\min}	Type of mechanism	(NP)Number of specified points through which the coupler point approximately passes.
1	50	50	90	13.56	Crank - rocker	5
2	50	50	90	1.852	Non Grashofian	3
3	50	50	90	13.07	Crank - rocker	4
4	50	50	90	5.366	Crank - rocker	9

Out of all the four solutions, the fourth one can be selected (see Plot 3.2). To observe the accuracy of the results, X_error (%) and Y_error (%) are introduced. These errors, at an i^{th} position (i.e. corresponding to θ_{2i}), are defined as

$$(\%) X_error [i] = [\{ (x_i)_g - (x_i)_s \} 100 / \{ (x_{\max})_s - (x_{\min})_s \}]$$

and

$$(\%) Y_error [i] = [\{ (y_i)_g - (y_i)_s \} 100 / \{ (y_{\max})_s - (y_{\min})_s \}],$$

where

$i = 1$ to n (no of specified points),

suffixes 'g' and 's' denote the generated and specified points,

x_{\max} = the maximum value of x_i (x _ coordinate of the point at the i^{th} position),

x_{\min} = the minimum value of x_i ,

y_{\max} = the maximum value of y_i (y _ coordinate of the point at the i^{th} position)

and y_{\min} = the minimum value of y_i .

To reduce the errors near the points 3 and 4 (as expected) weighting of the error function has been considered and after the first iteration a better solution (regarding those two points) is obtained (see Plot 3.3). The errors at all the points corresponding to θ_{2i} ($i = 1$ to n), before and after weighting the error functions, are given below.

		Before weighting the error function		After weighting the error function	
Point	θ_2	(%) X_error	(%) Y_error	(%) X_error	(%) Y_error
1	0	14.82	-15.19	6.22	-7.83
2	30	-2.65	-14.99	-2.50	-8.11
*3	60	21.41	14.21	14.11	10.30
*4	90	13.93	8.67	7.76	5.23
5	120	3.029	-2.19	-1.24	-0.96
6	150	-2.70	-2.58	-5.27	-5.38
7	180	-3.07	-4.73	-4.20	-6.99
8	210	-0.71	-3.62	-0.86	-5.10
9	240	-0.0002	0.0065	0.14	0.43
10	270	-3.28	3.925	-3.92	4.79
11	300	-7.34	5.414	-10.1	7.85
12	300	-1.63	1.039	7.83	5.485

The errors at the other points (e.g. 11 and 12) are now more. So applying weights is more useful when the structural error at some points are critical for a particular application.

3.2. FUNCTION GENERATION

If a 4R mechanism has to generate a particular function (which is given), with more than three precision positions, then the optimization method is more useful regarding the overall minimization of the error.

Let θ_{2-0i} and θ_{4-0i} be the input and output angles, respectively at the i^{th} position as shown in Fig 3.2. Now let us assume the initial positions of the input and output links, which are at angles θ_{2-0} and θ_{4-0} , respectively (Fig 3.2). Therefore, from Fig 3.2, for n design positions

$$\theta_{2i} = \theta_{2-0} + \theta_{2-0i} \quad (i = 1 \text{ to } n) \quad [3.2.1]$$

$$\text{and} \quad \theta_{4i} = \theta_{4-0} + \theta_{4-0i} \quad (i = 1 \text{ to } n). \quad [3.2.2]$$

Let ϕ_{4i} be the actual angular displacement of the output link of the synthesized mechanism corresponding to the input angle θ_{2i} . Writing the well known Freudenstein's equation ^[1] for the i^{th} position, we get

$$D_1 \cos \theta_{2i} - D_2 \cos \phi_{4i} + D_3 = \cos (\theta_{2i} - \phi_{4i}), \quad [3.2.3]$$

where

$$\begin{aligned} D_1 &= L_1 / L_4, \quad D_2 = L_1 / L_2 \\ \text{and} \quad D_3 &= (L_1^2 + L_2^2 - L_3^2 + L_4^2) / (2 L_2 L_4). \end{aligned} \quad [3.2.4]$$

If the angle ϕ_{4i} is replaced by θ_{4i} (which is available at θ_{2i}) then Equation 3.2.3 is not satisfied in general and the difference between the L.H.S. and the R.H.S. of Equation 3.2.3 gives a small error. While using the *least square technique*, let e be the error defined as

$$e = \sum_{i=1}^n [D_1 \cos \theta_{2i} - D_2 \cos \theta_{4i} + D_3 - \cos (\theta_{2i} - \theta_{4i})]^2 \quad [3.2.5]$$

Now, for the error e to be minimum, we take the partial derivatives of e with respect to D_1 , D_2 and D_3 separately, and set each of the derivatives to zero. Thus

$$\begin{aligned}\frac{\partial e}{\partial D_1} &= 2 \sum_{i=1}^n [D_1 \cos \theta_{2i} - D_2 \cos \theta_{4i} + D_3 - \cos(\theta_{2i} - \theta_{4i})] \cos \theta_{2i} \\ &= 0, \quad [3.2.6]\end{aligned}$$

$$\begin{aligned}\frac{\partial e}{\partial D_2} &= -2 \sum_{i=1}^n [D_1 \cos \theta_{2i} - D_2 \cos \theta_{4i} + D_3 - \cos(\theta_{2i} - \theta_{4i})] \cos \theta_{4i} \\ &= 0 \quad [3.2.7]\end{aligned}$$

$$\begin{aligned}\text{and } \frac{\partial e}{\partial D_3} &= 2 \sum_{i=1}^n [D_1 \cos \theta_{2i} - D_2 \cos \theta_{4i} + D_3 - \cos(\theta_{2i} - \theta_{4i})] \\ &= 0. \quad [3.2.8]\end{aligned}$$

The above equations are rearranged to obtain

$$\begin{aligned}D_1 \sum_{i=1}^n \cos^2 \theta_{2i} - D_2 \sum_{i=1}^n \cos \theta_{4i} \cos \theta_{2i} + D_3 \sum_{i=1}^n \cos \theta_{2i} \\ = \sum_{i=1}^n \cos(\theta_{2i} - \theta_{4i}) \cos \theta_{2i}, \quad [3.2.9]\end{aligned}$$

$$\begin{aligned}D_1 \sum_{i=1}^n \cos \theta_{2i} \cos \theta_{4i} - D_2 \sum_{i=1}^n \cos^2 \theta_{4i} + D_3 \sum_{i=1}^n \cos \theta_{4i} \\ = \sum_{i=1}^n \cos(\theta_{2i} - \theta_{4i}) \cos \theta_{4i} \quad [3.2.10]\end{aligned}$$

$$\begin{aligned}\text{and } D_1 \sum_{i=1}^n \cos \theta_{2i} - D_2 \sum_{i=1}^n \cos \theta_{4i} + D_3 \sum_{i=1}^n 1 \\ = \sum_{i=1}^n \cos(\theta_{2i} - \theta_{4i}) \quad [3.2.11]\end{aligned}$$

The above three equations are linear, simultaneous and non homogeneous in three unknowns, D_1 , D_2 and D_3 . Hence, these can be solved by using the Cramer's rule.

Finally from Equation 3.2.4 we get the link lengths. By changing the assumed values of θ_{2-0} and θ_{4-0} , we can reduce the absolute value of $(\theta_{4i} - \phi_{4i})$ at each design position and thus get a better solution.

Flow chart with an illustration - The programming for the optimized function generation problem can be easily understood from Chart 3.2. There are mainly two variables

(θ_{2-0} and θ_{4-0}) to be assumed and one more parameter L_1 is used to define a unique mechanism. An induced condition ($\text{Sqrt3} \geq 0$), mentioned in the Chart 3.2, can easily be satisfied. For obtaining a better solution, one should minimize $|\theta_{4-0} - \phi_{4i}|$ by changing the initially assumed values. After computing the link lengths, $\text{DRAW_4R}()$ is executed to animate the mechanism. For verification of the result we print $|\theta_{4i} - \phi_{4i}|$ at each θ_{2i} and one can also display the result graphically.

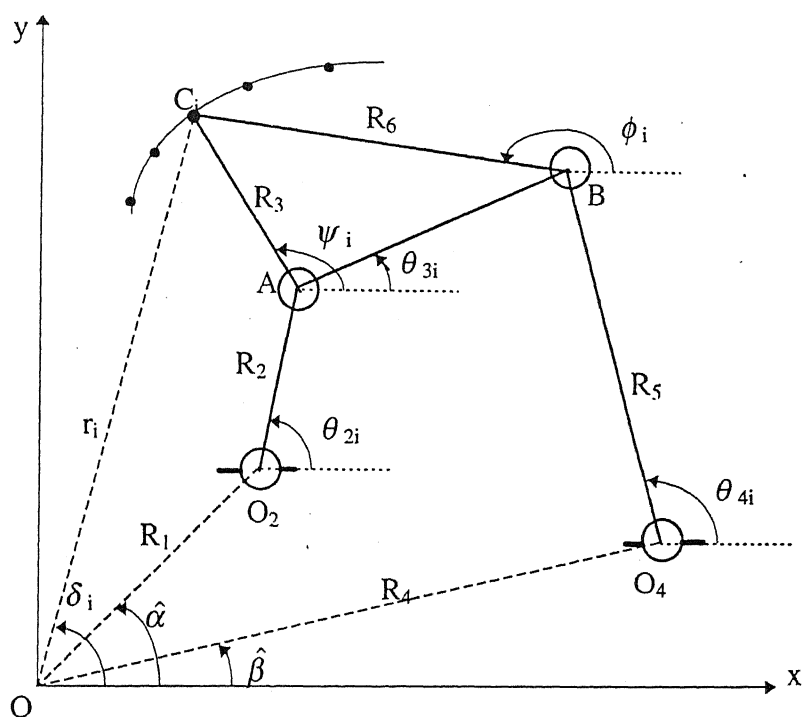
We have considered a problem for which a set of 8 coordinated input-output angles are given. These are given below. The angles are expressed in degrees.

i	1	2	3	4	5	6	7	8
θ_{2-0i}	15	60	105	150	195	240	285	330
θ_{4-0i}	30	50	70	90	110	100	85	75

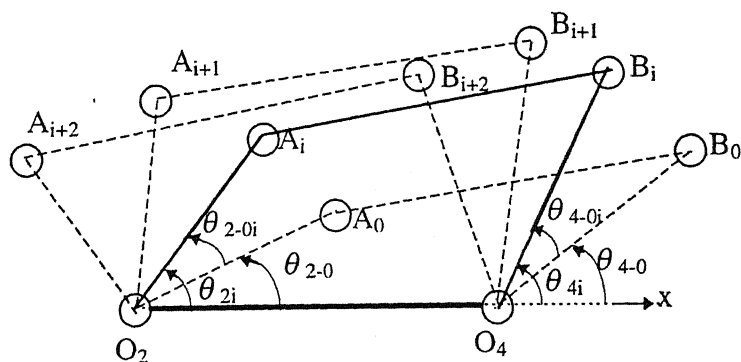
Assuming $\theta_{2-0} = 0.0$, $\theta_{4-0} = 0.0$ and $L_1 = 5$, the dimensions of the synthesized mechanism along with the plot of θ_2 vs. ϕ_4 are shown in Plot 3.4.

The errors (i.e., $|\theta_{4i} - \phi_{4i}|$) at different positions are given mentioned below.

i	1	2	3	4	5	6	7	8
θ_{2-0i}	15	60	105	150	195	240	285	330
$ \theta_{4i} - \phi_{4i} $	1	2	0	0	6	6	10	12



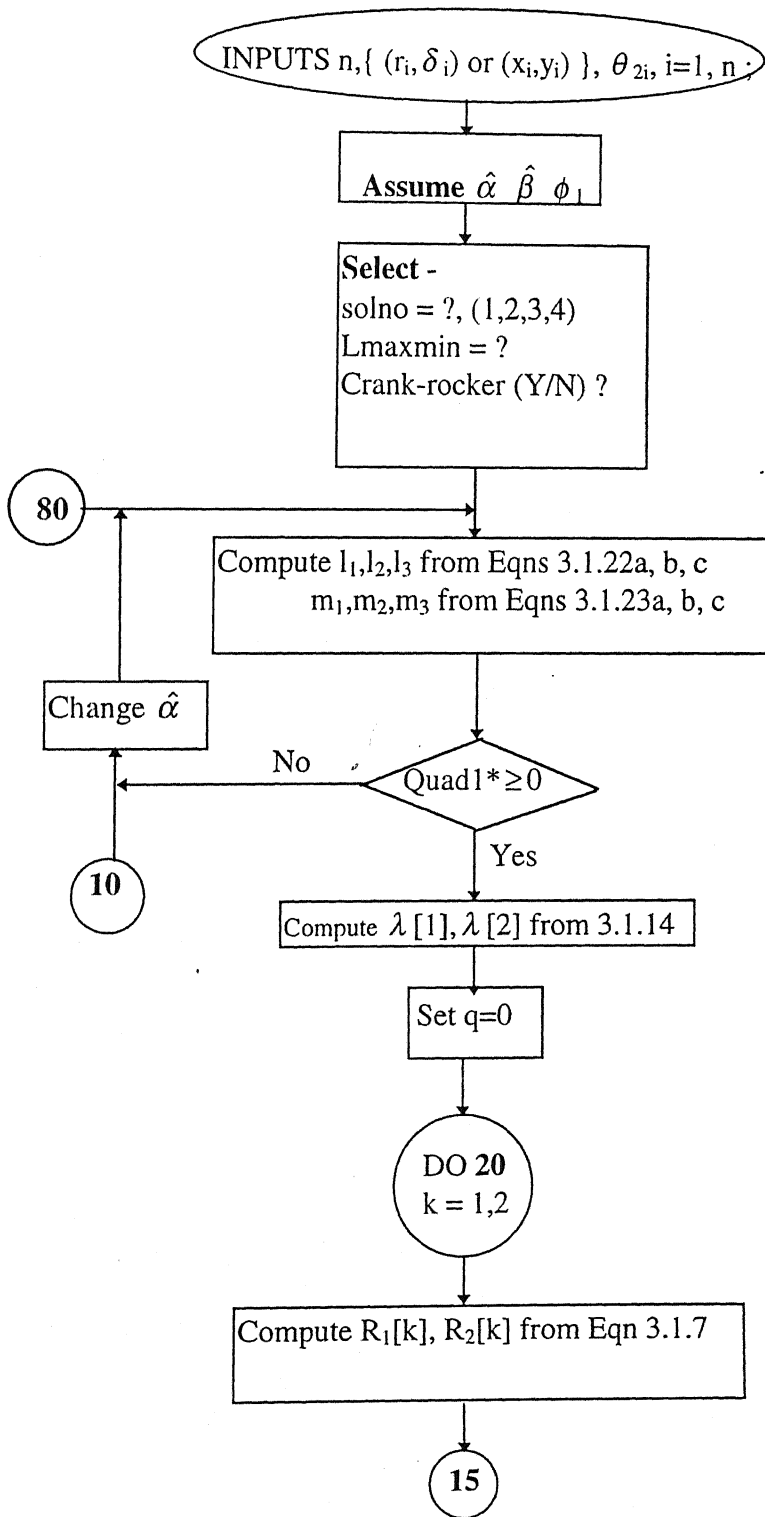
[Fig 3.1]

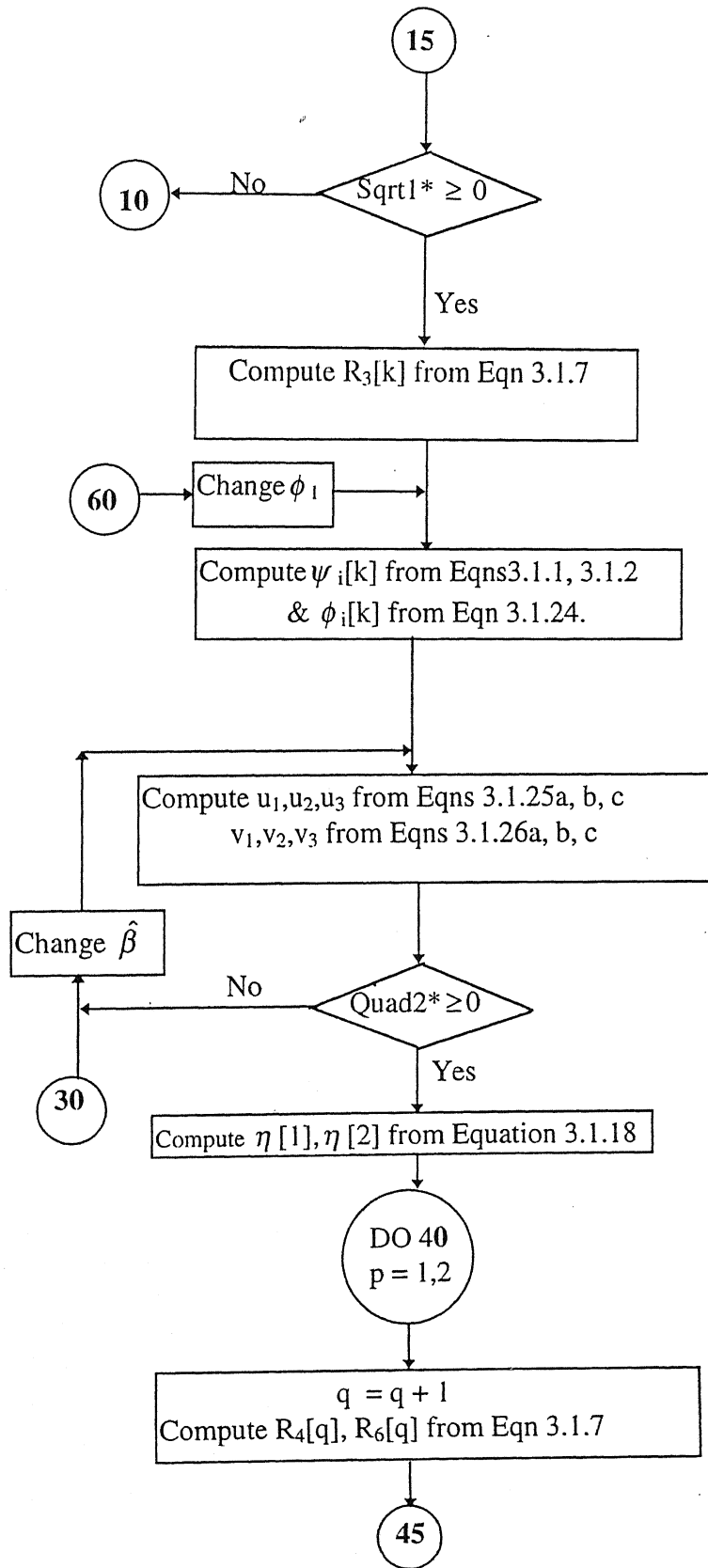


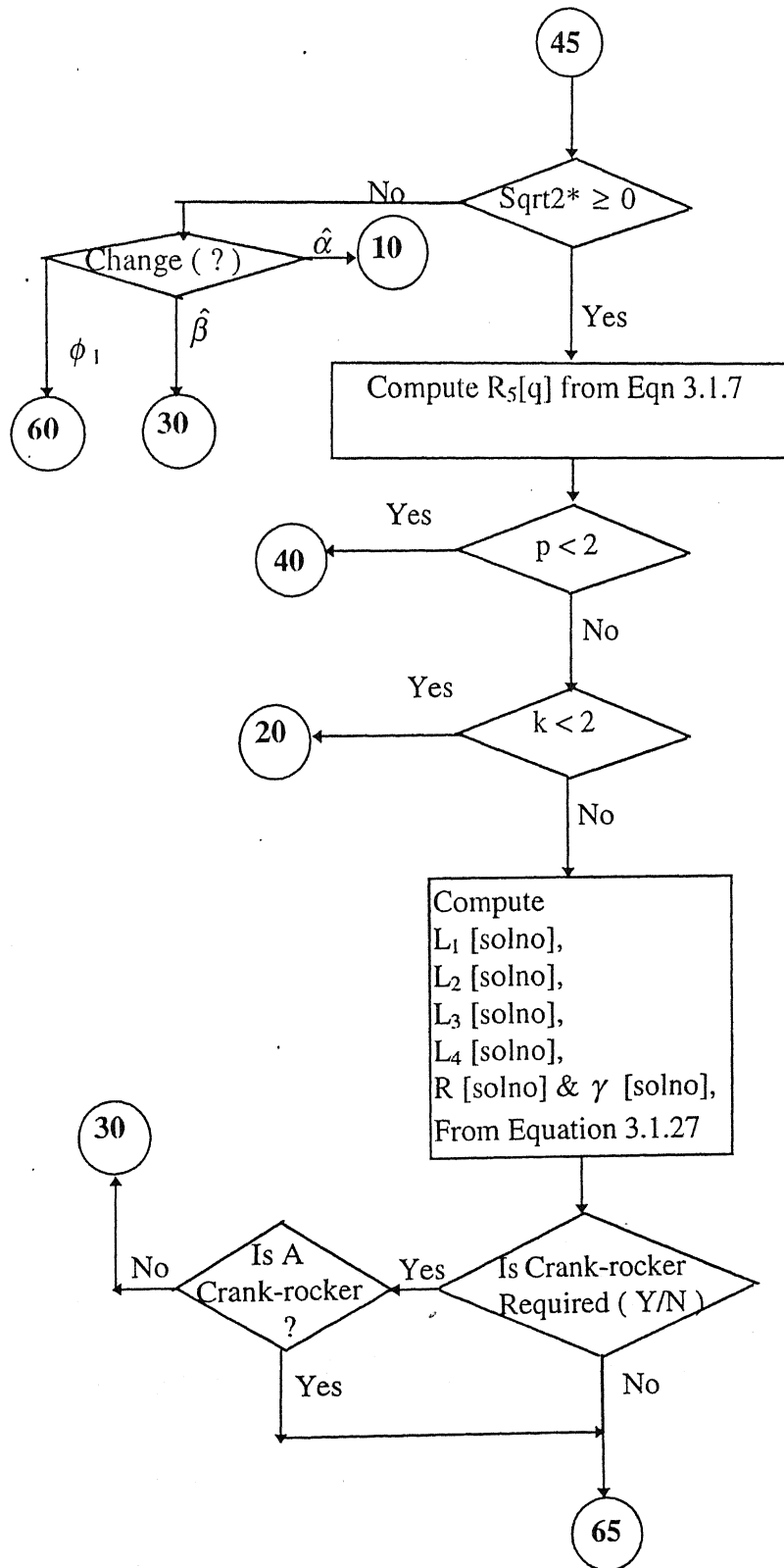
[Fig 3.2]

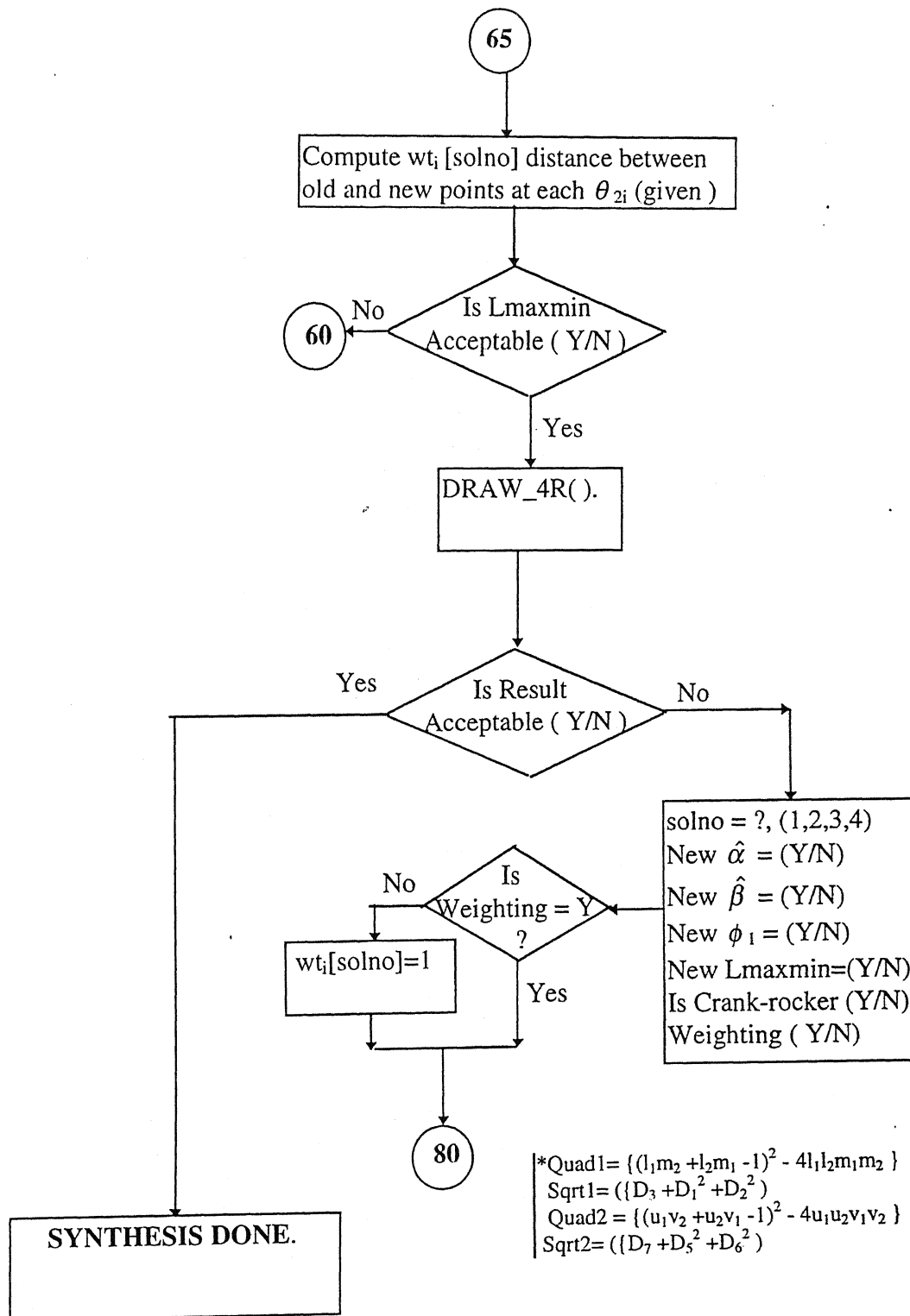
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[OPTIMIZATION APPROACH FOR PATH GENERATION]



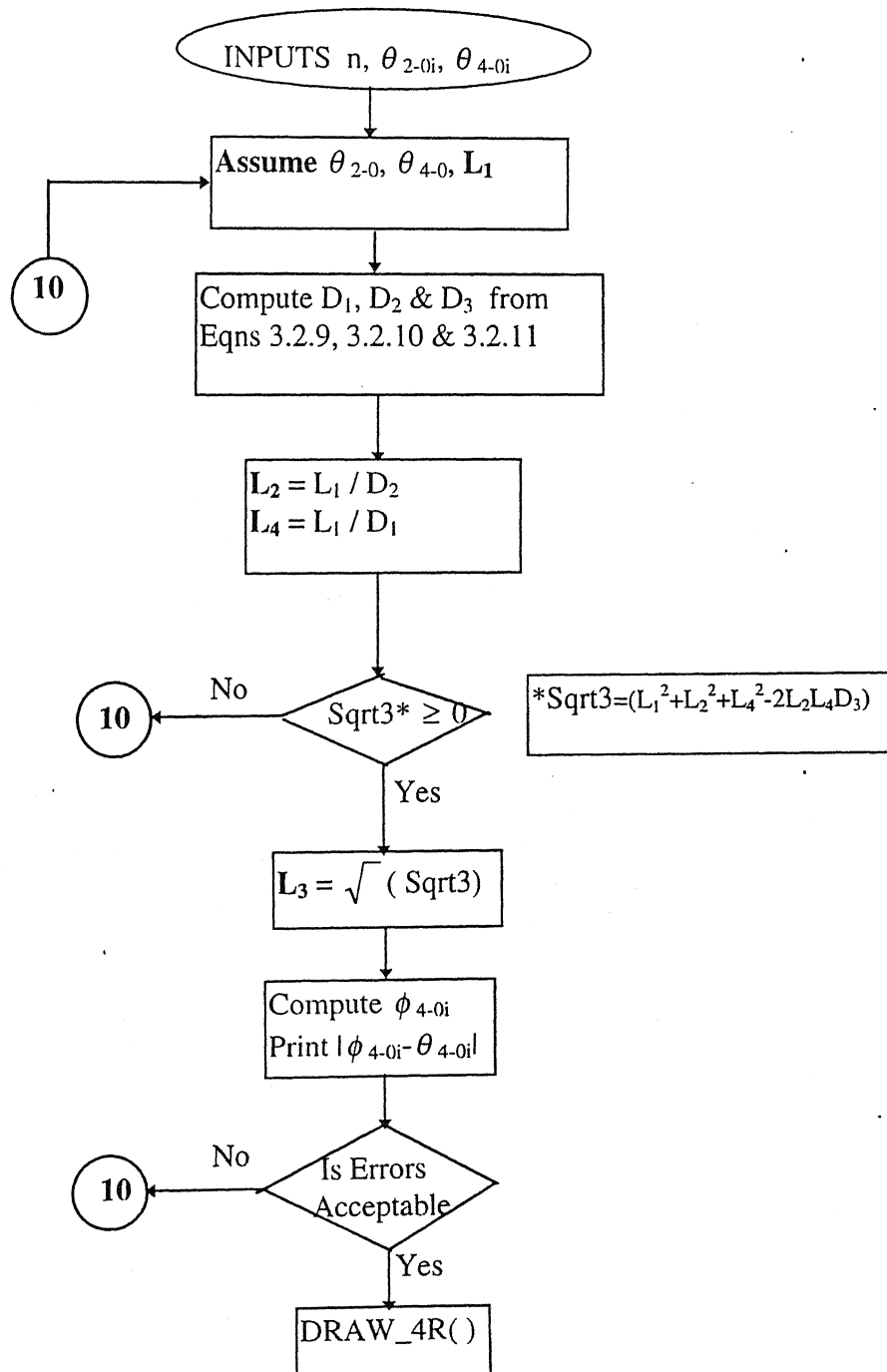




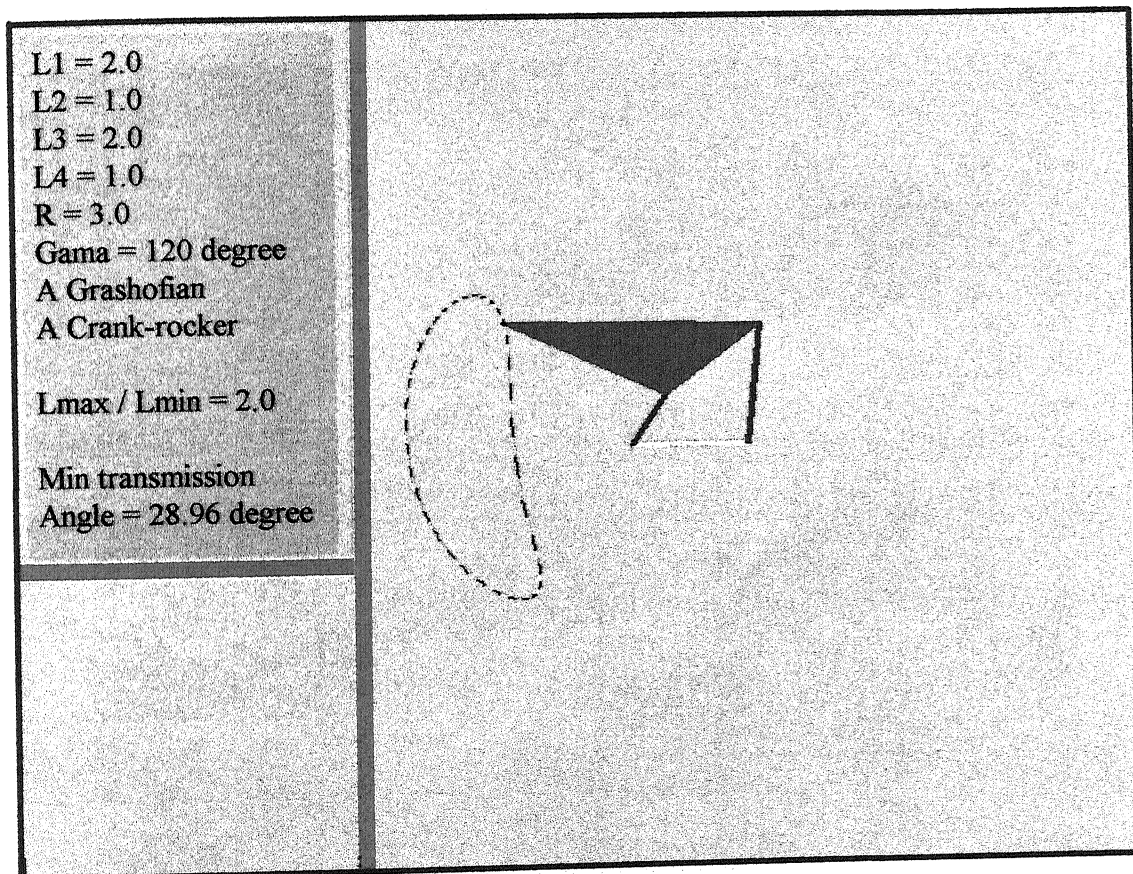


[Chart 3.1]

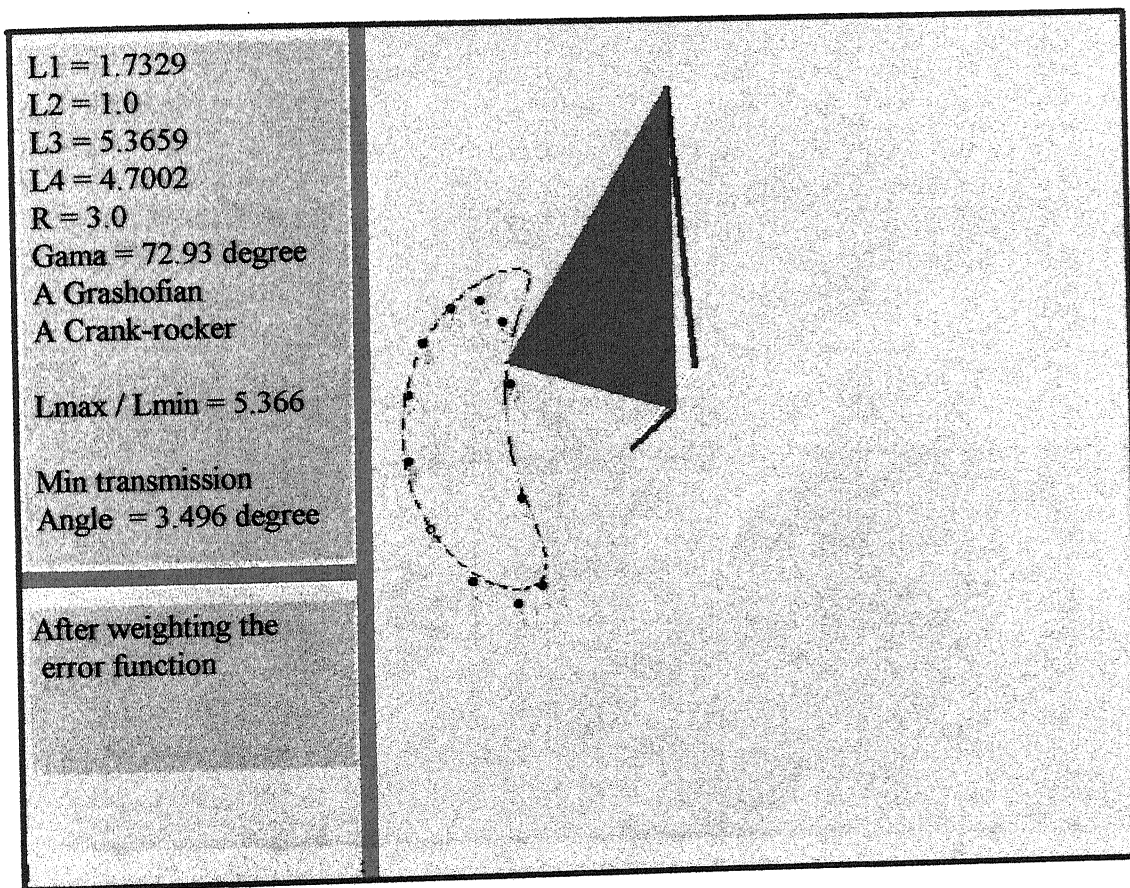
[OPTIMIZATION APPROACH FOR FUNCTION GENERATION]



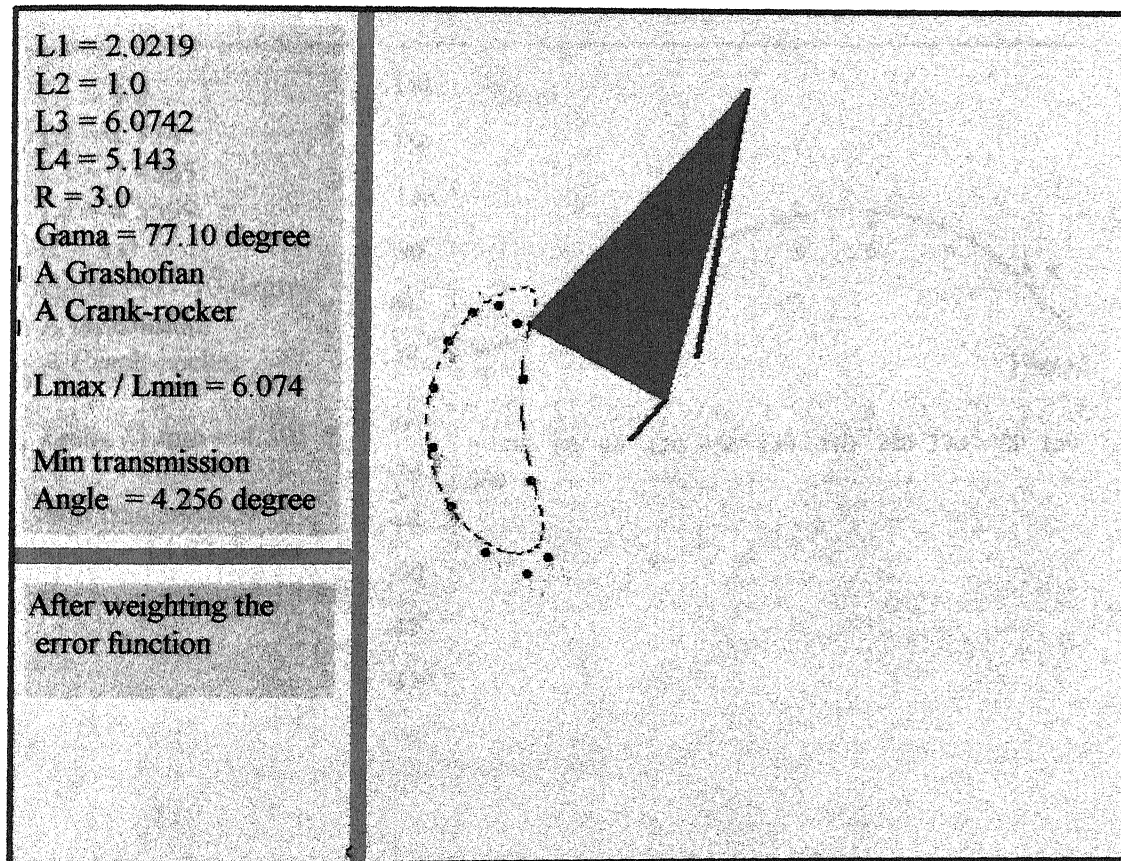
[Chart 3.2]



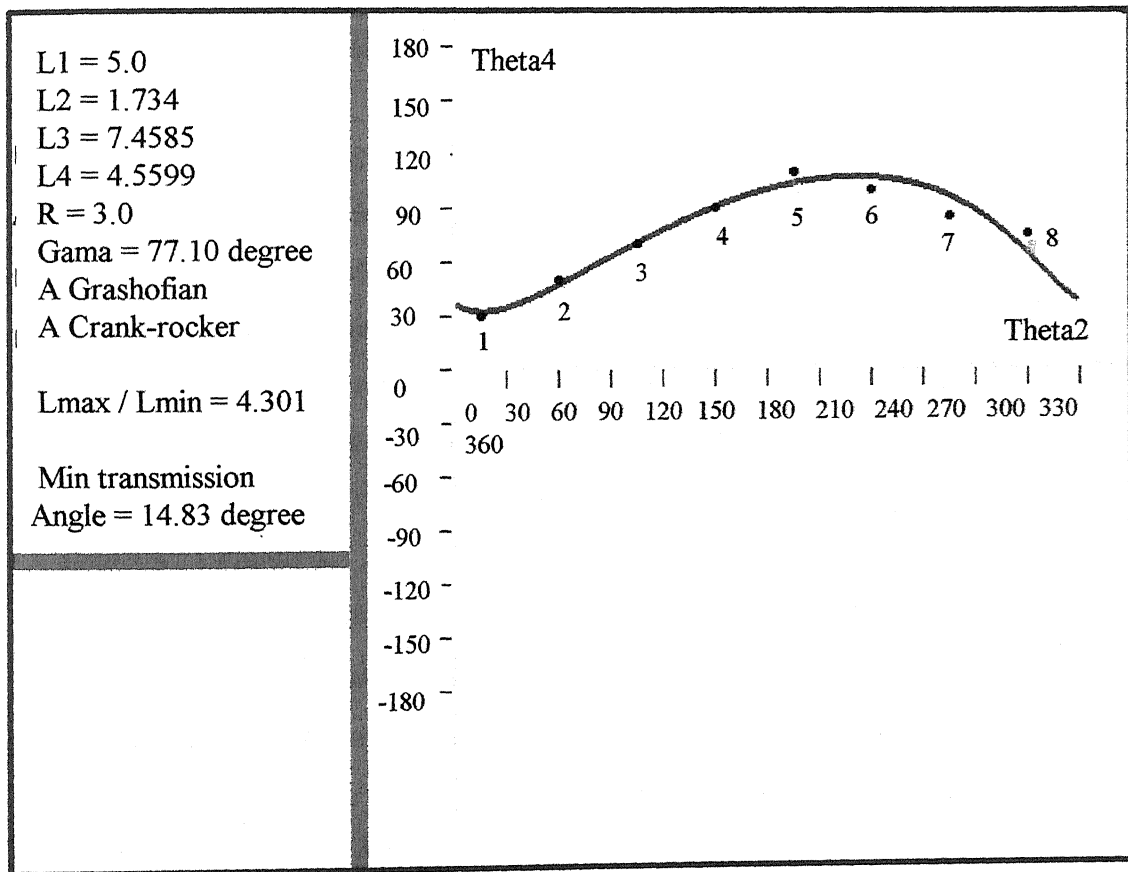
[Plot 3.1]



[Plot 3.2]



[Plot 3.3]



[Plot 3.4]

4. COUPLER CURVES

In this chapter, two important aspects of 4R coupler curves are presented. The first one is related to the generation of two cusps by a coupler point of a 4R mechanism. The second one is concerned with the coupler curve cognate linkages.

4.1 TWO CUSPS PROBLEM

The objective is to design a crank-rocker mechanism, in which a coupler point has two cusps at prescribed locations and the amount of crank rotation as the coupler point moves from one cusp to the another is also prescribed.

Before solving this problem, we should note the following result^[1].

For every crank-rocker mechanism, there exists a coupler point which has two cusps. This coupler point can be located at the instantaneous center corresponding to the configurations when the crank-pin A lies on the *Thales circle* CR_T (a circle with O_2O_4 as a diameter) (Fig 4.1).

It can be easily proved that the sum of the crank rotation (θ_{2c}) and the rocker rotation (θ_{4c}) as the coupler point goes from one cusp to the another is π . This can be proved from Fig 4.1 as explained below.

Since $\Delta O_4B_2A_2$ is obtained by rotating $\Delta O_4B_1A_1$ about O_4 through an angle θ_{4c} and $\angle B_2O_4B_1 = \angle C_2O_4C_1 = \theta_{4c}$, we get $\angle A_2O_4A_1 = \theta_{4c}$. But A_1, A_2, O_2 and O_4 are on the same circle CR_T , therefore

$$\theta_{2c} + \theta_{4c} = \pi \quad \text{or} \quad \theta_{4c} = \pi - \theta_{2c}. \quad [4.1.1]$$

Let $C_1 \equiv (x_1, y_1)$ and $C_2 \equiv (x_2, y_2)$ are the prescribed locations of the cusps whereas θ_{2c} is the given crank rotation as the coupler point goes from one cusp to the another.

It is also known, that the double point (if it exists on the coupler curve) lies on a circle

known as *circle of singular foci*^[1]. But, the circle of singular foci passes through the fixed hinges (i.e. O_2 and O_4). Hence we can say that C_1 , C_2 , O_2 and O_4 will lie on the same circle.

From Fig 4.1, concentrating on Δ 's $O_4A_1C_1$ and $O_4A_2C_2$, we observe that

$$\angle A_1O_4C_1 = \angle A_1O_4B_2 + \theta_{4c} = \angle A_2O_4C_2,$$

$$\angle C_1A_1O_4 = \angle C_2A_2O_4 = \pi / 2$$

and

$$O_4A_1 = O_4A_2.$$

Therefore, we can say that $\Delta O_4A_1C_1 \equiv \Delta O_4A_2C_2$ and $O_4C_1 = O_4C_2$.

Thus, for the synthesis of the desired mechanism, we draw a circle (CR_1) with the following characteristics :

- [i] It passes through C_1 and C_2 .
- [ii] The angle subtended by C_1C_2 at $O_4 = (\pi - \theta_{2c})$.
- [iii] $O_4C_1 = O_4C_2$.

In other words, we can draw a circle CR_1 passing through C_1 , C_2 and O_4 such that O_4 is on the mid-normal of C_1C_2 and the angle subtended by C_1C_2 at O_4 is $(\pi - \theta_{2c})$ (Fig 4.2).

Suppose

$C_0 \equiv (x_{C0}, y_{C0})$ is the center of CR_1 (of radius r_1),

$O_4 \equiv (x_{O4}, y_{O4})$,

p = the distance of C_0 from C_1C_2

and C_{12} = distance between C_1 and C_2 .

Therefore, from Fig 4.2

$$C_{12} = \sqrt{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}, \quad [4.1.2]$$

$$\delta \text{ (slope of } C_1C_2) = \tan^{-1} [(y_1 - y_2) / (x_1 - x_2)], \quad [4.1.3]$$

$$r_1 = C_{12} / (2 \sin \theta_{2c}), \quad [4.1.4]$$

$$p = \sqrt{r_1^2 - (C_{12}^2 / 4)}, \quad [4.1.5]$$

$$x_{C0} = x_2 + (C_{12} / 2) \cos \delta + p \sin \delta, \quad [4.1.6a]$$

$$y_{C0} = y_2 + (C_{12} / 2) \sin \delta - p \cos \delta, \quad [4.1.6b]$$

$$x_{O4} = x_2 + (C_{12} / 2) \cos \delta + (p + r_1) \sin \delta \quad [4.1.7a]$$

and

$$y_{O4} = y_2 + (C_{12} / 2) \sin \delta - (p + r_1) \cos \delta. \quad [4.1.7b]$$

Writing the equation of the circle CR_1 , we have

$$(x - x_{C0})^2 + (y - y_{C0})^2 = r_1^2. \quad [4.1.8]$$

Since $O_2 [\equiv (x_{O2}, y_{O2})]$ also lies on the same circle CR_1 , x_{O2} can be chosen anywhere on the circle between $(x_{C0} - r_1)$ and $(x_{C0} + r_1)$. Hence, from Equation 4.1.8 y_{O2} is known. Thus O_2 and O_4 are known, and we can draw the *Thales circle* CR_T .

Let $C_T [\equiv (x_{CT}, y_{CT})]$ and r_t be the center and radius of the circle CR_T , respectively.

$$\text{Therefore, } x_{CT} = (x_{O2} + x_{O4}) / 2, \quad [4.1.9a]$$

$$y_{CT} = (y_{O2} + y_{O4}) / 2 \quad [4.1.9b]$$

and

$$r_t = [\sqrt{ \{ (x_{O4} - x_{O2})^2 + (y_{O4} - y_{O2})^2 \} }] / 2. \quad [4.1.10]$$

Equation of the circle CR_T is given by

$$(x - x_{CT})^2 + (y - y_{CT})^2 = r_t^2. \quad [4.1.11]$$

The equation of the line O_2C_1 is

$$y = m x + c, \quad [4.1.12]$$

where

$$m = (y_1 - y_{O2}) / (x_1 - x_{O2}) \quad [4.1.13a]$$

and

$$c = y_{O2} - m x_{O2} \quad [4.1.13b]$$

We know that the crank-pin lies on the point of intersection of the crank circle and the Thales circle when the coupler point lies at either of the cusps. A cusp is an instantaneous center of velocity of the coupler. Also for a 4R mechanism, the instantaneous center of the coupler relative to the fixed link O_2O_4 lies at the intersection of lines drawn through the input and the output links (using the well-known *theorem of three centers*). In other words, at the configuration when the coupler point lies at the instantaneous center (or cusp on the coupler curve), the crank-pin lies on the line joining the fixed end of the crank and the instantaneous center (or cusp on the coupler curve). On this basis, we can say that the crank-pins A_1 [say (x_{A1}, y_{A1})] and A_2 [say (x_{A2}, y_{A2})] (lying on the Thales circle CR_T) lie on the lines O_2C_1 and O_2C_2 respectively (Fig 4.2).

Now, from Equations 4.1.11 and 4.1.12

$$(x - x_{CT})^2 + (m x + c - y_{CT})^2 = r_t^2$$

or

$$(1 + m^2) x^2 + 2(t_1 + m t_2) x + (t_1^2 + t_2^2 - r_t^2) = 0, \quad [4.1.14]$$

where

$$t_1 = -x_{CT} \quad \text{and} \quad t_2 = c - y_{CT}.$$

The quadratic Equation 4.1.14 gives two values of x , as the line O_2C_1 cuts the circle CR_T at two points. Solving this equation, we get the x coordinate of A_1 i.e. x_{A1} . Then, from

Equations 4.1.12, 4.1.13a and 4.1.13b we can find the y _ coordinate of A_1 i.e. y_{A1} . Hence A_1 is known.

To find all the link lengths, we have to locate the point B_1 [say (x_{B1} , y_{B1})]. Since the point B_1 (the free end of the follower when the coupler point lies at the cusp C_1) lies anywhere on the line O_4C_1 , Therefore, we can chose x_{B1} between x_1 (the x _ coordinate of C_1) and x_{O4} (the x _ coordinate of O_4).

Now the equation of line O_4C_1 is written as

$$y = m_1 x + c_1, \quad [4.1.15]$$

where

$$m_1 = (y_1 - y_{O4}) / (x_1 - x_{O4}) \quad [4.1.16a]$$

and

$$c_1 = y_{O4} - m_1 x_{O4}. \quad [4.1.16b]$$

Therefore, we can find y_{B1} corresponding to x_{B1} on the line O_4C_1 (from Equation 4.1.15). Depending upon the selection of x_{O2} and x_{B1} we can find many solutions but we control these selections to get a crank-rocker mechanism with O_2A_1 as the shortest link. Thus a crank-rocker mechanism $O_2A_1B_1O_4$ is designed.

Flow chart with an illustration - Chart 4.1 clearly explains the steps used to design a crank-rocker mechanism for the present problem. The assumptions made are

$$x_{O2} = x_{C0} - r_1/2$$

$$\text{and } x_{B1} = (x_1 + x_{O4})/2,$$

which can be changed to get a required mechanism. As an illustration, we have selected two cusps at

$$C_1 \equiv (10, 8),$$

$$C_2 \equiv (1, 1)$$

$$\text{and } \theta_{2c} = 120^0.$$

The designed mechanism with coupler curve and link lengths are shown in Plot 4.1.

4.2 COGNATE LINKAGES

The term *cognate* (coined by Hartenberg and Denavit) of a mechanism means a different mechanism which generates the same coupler curve. The most important theorem^[1], developed by Samuel Roberts (1875) and Chebyshev (1878) independently, states that *three different planar 4R linkages trace identical coupler curves*.

Let O_2ABO_4 is a given crank-rocker mechanism and C is its coupler point (Fig 4.3). There exist two 4R cognate linkages of this mechanism. To get the cognate linkages, first draw parallelograms O_2ACD and O_4BCG . On sides DC and CG, make triangles EDC and FCG respectively similar to the triangle CAB. Again draw a parallelogram OEFC and make the hinge O fixed. This ten-link mechanism is found^[1] constrained (with degree of freedom = 1). Thus, the common coupler point C of mechanisms O_2ABO_4 , O_2DEO and O_4FGO_4 generates the same coupler curve. The mechanism O_2DEO is called the *left-hand cognate* and O_4FGO_4 the *right-hand cognate*.

It should be noted ^[1] that, ΔOO_2O_4 is also similar to ΔCAB .

Now, suppose

$$O_2O_4 = L_1, O_2A = L_2, AB = L_3,$$

$$O_4B = L_4, AC = R \text{ and } BC = S.$$

The unknown design parameters of both cognates can be found analytically in the following manner.

[i] For O_4FCGO_4 (Right-hand cognate) - The design parameters RG_{L_1} , RG_{L_2} , RG_{L_3} , RG_{L_4} , RG_R and RG_S are given as below.

From Fig 4.3,

$$RG_{L_1} = OO_4 = (O_2O_4) (BC / AB)$$

(from similar Δ 's OO_2O_4 and CAB),

$$\text{or } RG_{L_1} = L_1 (S / L_3), \quad [4.2.1]$$

$$RG_{L_2} = OF = EC = LG_S = (S / L_3) L_2, \quad [4.2.2]$$

$$\begin{aligned}
 & RG_{L_3} = FG = (\sin \gamma_1 / \sin \gamma_3) GC \quad (\text{from } \Delta CDE) \\
 \text{or} \quad & RG_{L_3} = (S / L_3) L_4 \quad (\text{from } \Delta ABC), \\
 & \quad \quad \quad [4.2.3]
 \end{aligned}$$

$$RG_{L_4} = O_4G = S, \quad [4.2.4]$$

$$\begin{aligned}
 & RG_R = FC = (\sin \gamma_2 / \sin \gamma_3) GC \quad (\text{from } \Delta CDE), \\
 \text{or} \quad & RG_R = (R / L_3) L_4 \quad (\text{from } \Delta ABC), \\
 & \quad \quad \quad [4.2.5]
 \end{aligned}$$

and

$$RG_S = GC = L_4. \quad [4.2.6]$$

[ii] For O_2DCEO (Left-hand cognate) - The design parameters LG_{L_1} , LG_{L_2} , LG_{L_3} , LG_{L_4} , LG_R and LG_S are given as below.

From Fig 4.3,

$$\begin{aligned}
 LG_{L_1} &= O_2O = (O_2O_4) (AC / AB) \\
 & \quad \quad \quad (\text{from similar } \Delta \text{'s } OO_2O_4 \text{ and } CAB),
 \end{aligned}$$

$$\text{or} \quad LG_{L_1} = L_1 (R / L_3), \quad [4.2.7]$$

$$LG_{L_2} = O_2D = AC = R, \quad [4.2.8]$$

$$\begin{aligned}
 & LG_{L_3} = DE = (\sin \gamma_2 / \sin \gamma_3) DC \quad (\text{from } \Delta CDE) \\
 \text{or} \quad & LG_{L_3} = (R / L_3) L_2 \quad (\text{from } \Delta ABC), \\
 & \quad \quad \quad [4.2.9]
 \end{aligned}$$

$$LG_{L_4} = OE = FC = RG_R = (R / L_3) L_4, \quad [4.2.10]$$

$$LG_R = DC = O_2A = L_2 \quad [4.2.11]$$

and

$$LG_S = EC = (\sin \gamma_1 / \sin \gamma_3) DC \quad (\text{from } \Delta CDE)$$

$$\text{or } LG_S = (S / L_3) L_2 \quad (\text{from } \Delta ABC).$$

[4.2.12]

One can easily prove that the maximum-minimum link length ratio (L_{\max} / L_{\min}) for the original and its cognate linkages are same.

We can analyze the angular relationships of the various links of different cognate linkages in the following manner.

Observing the parallelograms O_2ACD , O_4BCG and $OECF$ we can say that

<u>Original Linkage</u>	<u>Left - Hand Cognate</u>	<u>Right - Hand Cognate</u>
Angular motion of link 2 \equiv	Angular motion of link 6 \equiv	Angular motion of link 8
Angular motion of link 3 \equiv	Angular motion of link 5 \equiv	Angular motion of link 10
Angular motion of link 4 \equiv	Angular motion of link 7 \equiv	Angular motion of link 9 .

Considering the above relationships we can conclude that, if the original linkage (O_2ABO_4) is a crank-rocker with say 2, as the crank (when 3 and 4 must oscillate), then the right-hand cognate is also a crank-rocker, with 8 as the crank ; however, the left-hand cognate then must be a double-rocker (with the coupler 6 undergoing the full rotation). Similarly both the cognates of a double-rocker linkage (say O_2DCEO) are crank-rockers ($OFCGO_4$ and O_2ACBO_4).

It should be noted that the transmission angle (μ_{\min}) for crank-rocker cognate linkages are same.

From Equation 2.2.1, the transmission angle for the mechanism is

$$\mu = \cos^{-1} [\{ (L_3^2 + L_4^2) - (L_1^2 + L_2^2) - 2 L_1 L_2 \cos \theta_2 \} / (2 L_3 L_4)].$$

[4.2.13]

The transmission angle for the right-hand cognate is given by (replacing L's by RG_L's)

$$\mu_{\text{right}} = \cos^{-1} [\{ (RG_L_3^2 + RG_L_4^2) - (RG_L_1^2 + RG_L_2^2) - 2 RG_L_1 RG_L_2 \cos \theta_{2r} \} / (2 RG_L_3 RG_L_4)]$$

where θ_{2r} represents the rotation of link 8.

Substituting the values of RG_L_1 , RG_L_2 , RG_L_3 and RG_L_4 in the above equation (from Equations 4.2.1 - 4.2.4), we get

$$\mu_{\text{right}} = \cos^{-1} [[\{ (L_4 S / L_3)^2 + S^2 \} - \{ (L_1 S / L_3)^2 + (L_2 S / L_3)^2 \} - 2 (L_1 S / L_3) (L_2 S / L_3) \cos \theta_{2r}] / \{ (L_4 S / L_3) S \}]$$

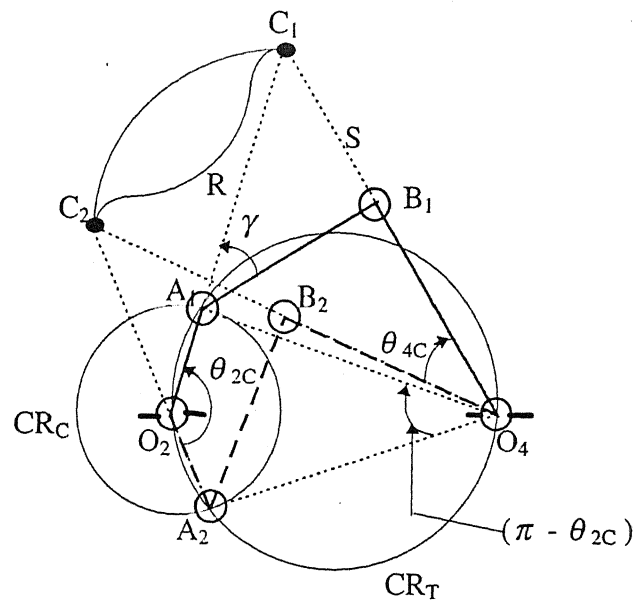
or

$$\mu_{\text{right}} = \cos^{-1} [\{ (L_3^2 + L_4^2) - (L_1^2 + L_2^2) - 2 L_1 L_2 \cos \theta_{2r} \} / (2 L_3 L_4)].$$

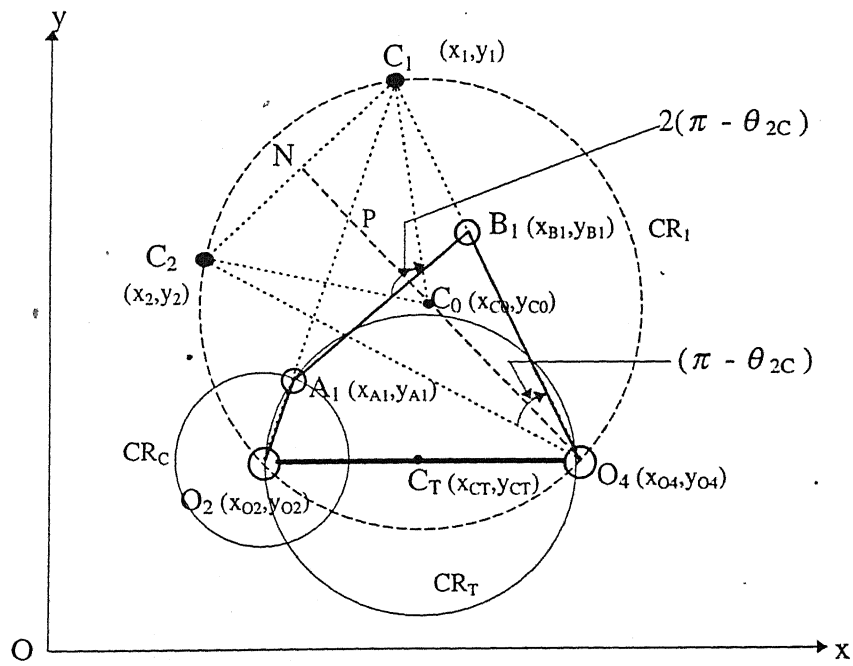
[4.2.14]

Suppose the original mechanism (O_2ABO_4) is a crank-rocker. Since, link 8 is equivalent to link 2 regarding the angular motion, we can say that θ_{2r} and θ_2 possess the same values for calculation of the minimum transmission angle. Hence, from Equations 4.2.13 and 4.2.14, μ_{right} and μ are same.

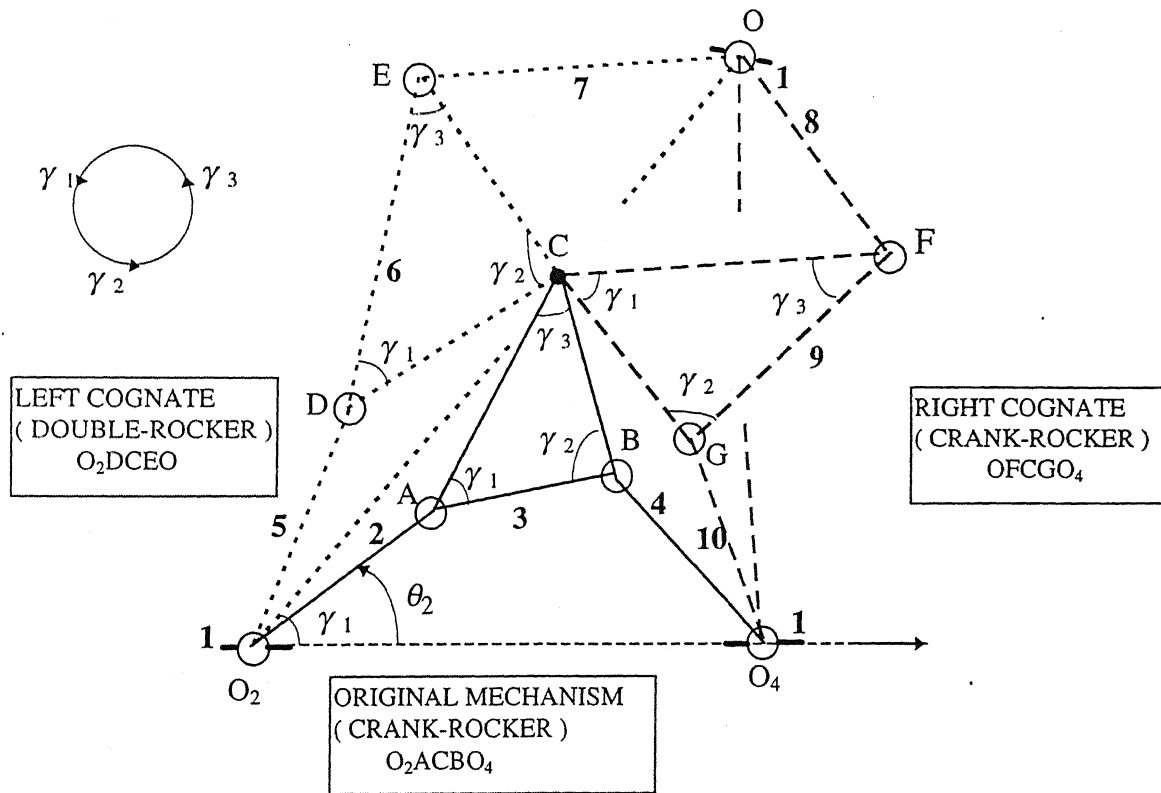
Illustration - For every crank-rocker and Grashofian double-rocker mechanisms, we can visualize the other two cognates using the present software. While solving a path generation problem for three precision points, we get a double-rocker mechanism, but as per the requirement of a crank-rocker mechanism one can select one of its cognates. Plot 4.2 shows the original mechanism (double-rocker) and its cognate linkages (crank-rockers) along with the coupler curve passing through the specified points in a path generation problem.



[Fig 4.1]

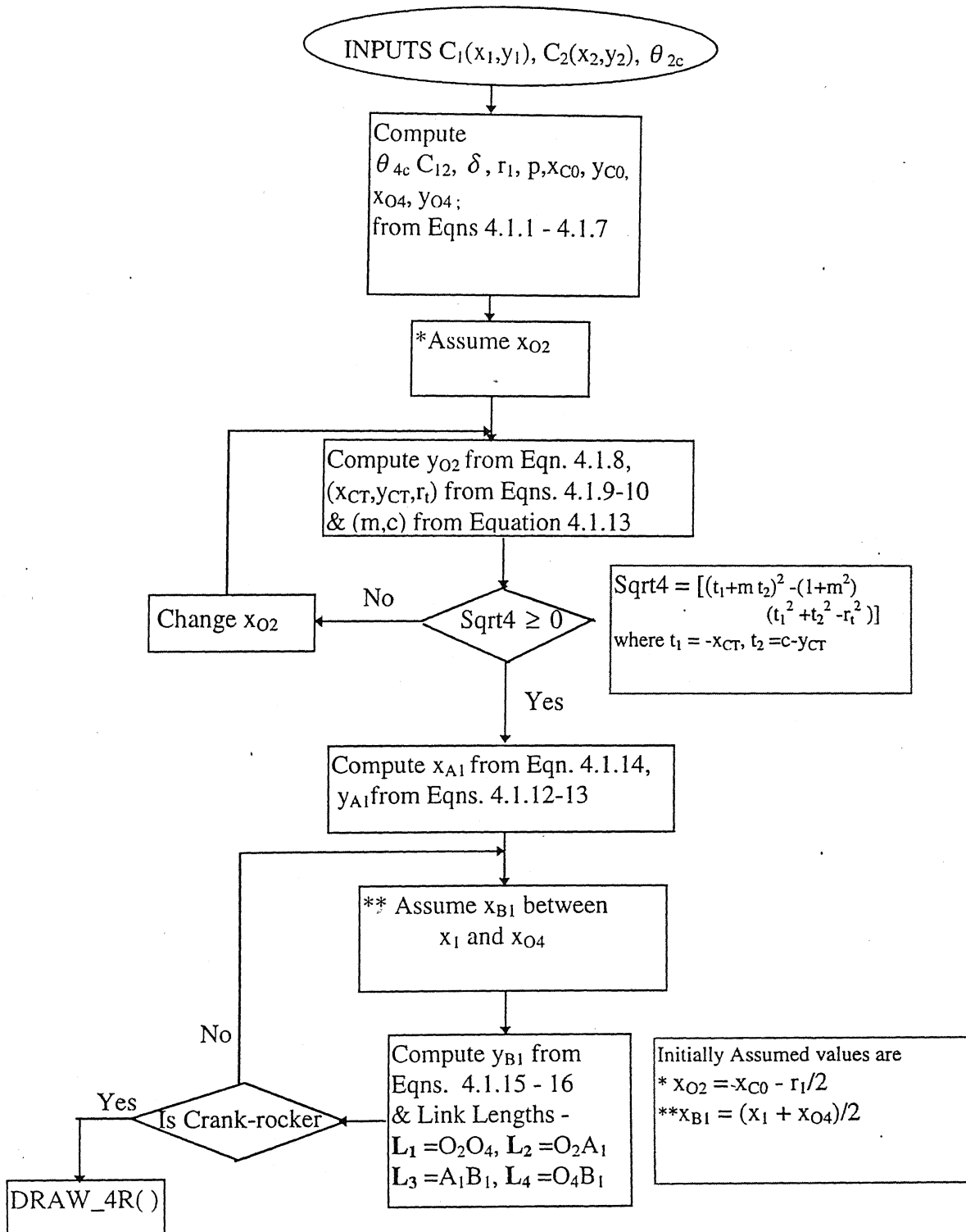


[Fig 4.2]

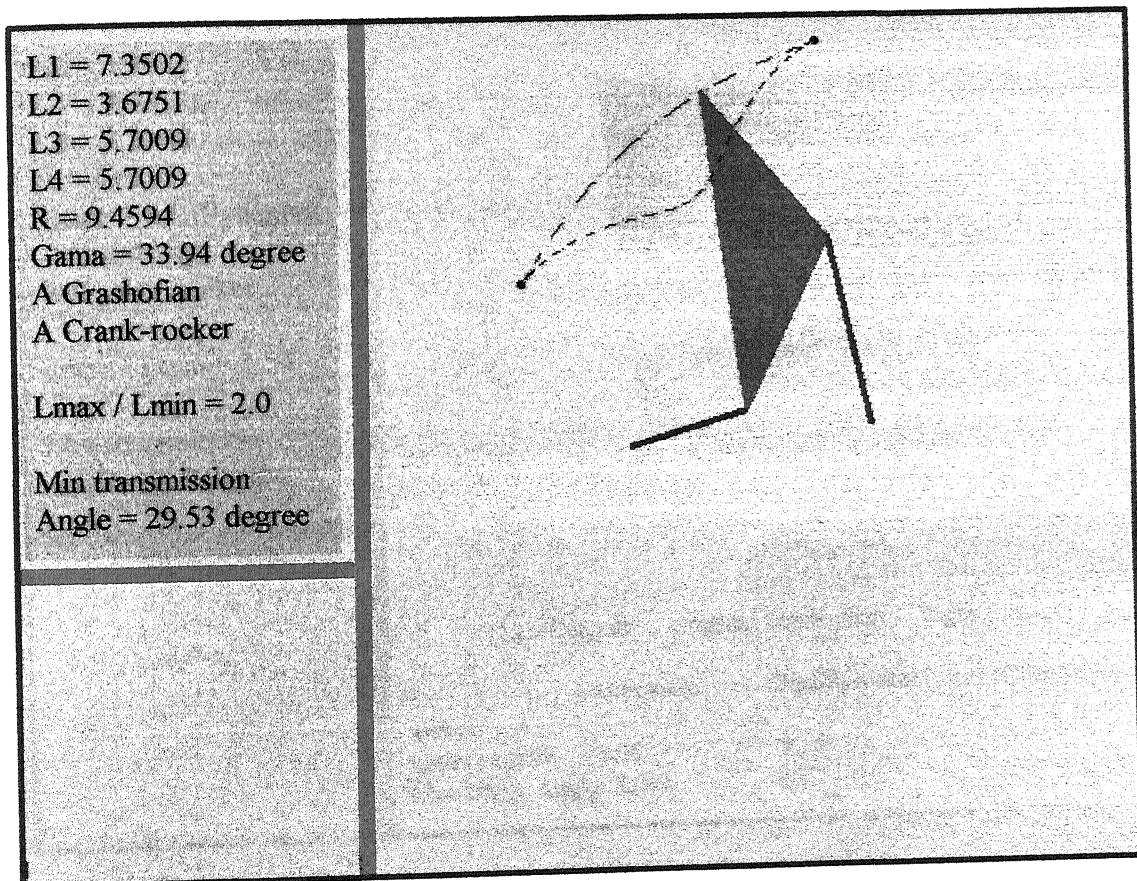


[Fig 4.3]

[TWO CUSPS PROBLEMS]

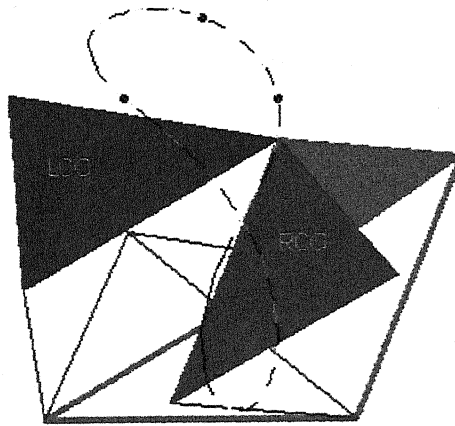


[Chart 4.1]



[Plot 4.1]

$L1 = 4.0$
 $L2 = 3.7612$
 $L3 = 2.531$
 $L4 = 3.5503$
 $R = 1.6259$
 $\text{Gama} = 65.19 \text{ degree}$
 A Grashofian
 A Double-rocker
 $L_{\max} / L_{\min} = 1.58$
 Min transmission
 Angle = 0.0 degree



	Left Cognate	Original Mechanism	Right Cognate
	Crank-rocker	Double-rocker	Crank-rocker
L_{\max} / L_{\min}	1.58	1.58	1.58
Min. trans. Angle	22.92	0.0	22.92

[Plot 4.2]

5. KINEMATIC CONDITIONS

In this chapter, we are going to present the synthesis of 4R mechanisms for prescribed kinematic parameters (e.g. angular velocities and angular accelerations etc.) of some of its links. Let θ_i , ω_i and α_i are the angular displacement, angular velocity and angular acceleration of the i^{th} link as shown in Fig 5.1. Depending upon the values of i we can have two cases.

Case 1. If θ_i , ω_i and α_i for input ($i = 2$) and output ($i = 4$) links are given.

Using Freudenstein 's equation we have

$$D_1 \cos \theta_2 + D_2 (-\cos \theta_4) + D_3 = \cos (\theta_2 - \theta_4), \quad [5.1]$$

where

$$D_1 = L_1 / L_4,$$

$$D_2 = L_1 / L_2$$

and $D_3 = (L_1^2 + L_2^2 - L_3^2 + L_4^2) / (2 L_2 L_4)$

[5.2]

with L_i as the length of the i^{th} link, $i = 1$ to 4.

Differentiating Equation 5.1 with respect to time, we get

$$D_1 (\omega_2 \sin \theta_2) + D_2 (-\omega_4 \sin \theta_4) = (\omega_2 - \omega_4) \{ \sin (\theta_2 - \theta_4) \} \quad [5.3]$$

and differentiating Equation 5.3 once more with respect to time, we obtain

$$\begin{aligned} D_1 (\omega_2^2 \cos \theta_2 + \alpha_2 \sin \theta_2) + D_2 (-\omega_4^2 \cos \theta_4 - \alpha_4 \sin \theta_4) \\ = (\omega_2 - \omega_4)^2 \{ \cos (\theta_2 - \theta_4) \} + (\alpha_2 - \alpha_4) \{ \sin (\theta_2 - \theta_4) \}. \end{aligned} \quad [5.4]$$

Three design parameters, D_1 , D_2 and D_3 can be solved from Equations 5.1, 5.3, and 5.4 using Cramer's rule. Then using Equation 5.2, and assuming any one of the link-lengths (i.e., the scale of the link diagram) we can have the other three link-lengths.

Case 2. If ω_i and α_i for input ($i = 2$), coupler ($i = 3$) and output ($i = 4$) links are given.

Writing the horizontal and vertical components of the loop closure equation for the loop $O_2ABO_4O_2$, we get

$$L_2 \cos \theta_2 + L_3 \cos \theta_3 = L_1 + L_4 \cos \theta_4 \quad [5.5]$$

and

$$L_2 \sin \theta_2 + L_3 \sin \theta_3 = L_4 \sin \theta_4. \quad [5.6]$$

Equations 5.5 and 5.6 can be combined into a complex exponential form as given below.

$$L_2 \exp(i \theta_2) + L_3 \exp(i \theta_3) = L_1 + L_4 \exp(i \theta_4)$$

or

$$(L_2/L_1) \exp(i \theta_2) + (L_3/L_1) \exp(i \theta_3) - (L_4/L_1) \exp(i \theta_4) = 1$$

or

$$Z_2 + Z_3 - Z_4 = Z_1 \quad [5.7]$$

where $Z_n = (L_n/L_1) \exp(i \theta_n)$, $n = 1, 2, 3, 4$ is a complex variable. For the fixed link $\theta_1 = 0$.

Differentiating Equation 5.7 successively with respect to time, we get

$$\omega_2 Z_2 + \omega_3 Z_3 - \omega_4 Z_4 = 0 \quad [5.8]$$

and

$$(\alpha_2 + i \omega_2^2) Z_2 + (\alpha_3 + i \omega_3^2) Z_3 - (\alpha_4 + i \omega_4^2) Z_4 = 0$$

or

$$V_2 Z_2 + V_3 Z_3 - V_4 Z_4 = 0 \quad [5.9]$$

where $V_n = (\alpha_n + i \omega_n^2)$, $n = 2, 3, 4$ is a complex variable.

From Equations 5.7 and 5.8, we get

$$\omega_{24} Z_2 + \omega_{34} Z_3 + \omega_4 Z_1 = 0 \quad [5.10]$$

and similarly, from Equations 5.7 and 5.9, we obtain

$$V_{24} Z_2 + V_{34} Z_3 + V_4 Z_1 = 0 \quad [5.11]$$

where $\omega_{ij} = \omega_i - \omega_j$ and $V_{ij} = V_i - V_j$.

Eliminating Z_2 , from Equations 5.10 and 5.11, we have

$$Z_3 = [\{ -V_4 + (\omega_4 / \omega_{24}) V_{24} \} / \{ V_{34} - (\omega_{34} / \omega_{24}) V_{24} \}] Z_1. \quad [5.12]$$

Then, from Equation 5.10

$$Z_2 = - [(\omega_4 / \omega_{24}) Z_1 + (\omega_{34} / \omega_{24}) Z_3] \quad [5.13]$$

and from Equation 5.7

$$Z_4 = -Z_1 + Z_2 + Z_3. \quad [5.14]$$

Thus knowing Z_2 , Z_3 and Z_4 (as a function of Z_1), we can draw the mechanism assuming a suitable value for one of the link-lengths.

Flow chart with illustrations - The steps used in the programming are clear from Chart 5.1. After computing the link-lengths, one can use the same software for its kinematic analysis for verifying the results.

For illustration of case 1, we have taken the following data's to design a mechanism.

$$(\theta_2 = 90^\circ, \omega_2 = 2 \text{ rad / sec and } \alpha_2 = 0 \text{ rad / sec}^2),$$

$$(\theta_4 = 60^\circ, \omega_4 = 3 \text{ rad / sec and } \alpha_4 = -1 \text{ rad / sec}^2)$$

and $L_1 = 10$ unit.

The results obtained, are

$$L_2 = 26.602 \text{ unit,}$$

$$L_3 = 23.394 \text{ unit}$$

$$\text{and } L_4 = 13.654 \text{ unit ;}$$

which form a non Grashofian mechanism. The angular displacements and velocities can be verified from Plot 5.1 and Plot 5.2, respectively.

For case 2, we have considered a problem with

$$(\omega_2 = 10 \text{ rad / sec and } \alpha_2 = 40 \text{ rad / sec}^2),$$

$$(\omega_3 = 4 \text{ rad / sec and } \alpha_3 = 60 \text{ rad / sec}^2),$$

$$(\omega_4 = 20 \text{ rad / sec and } \alpha_2 = 0 \text{ rad / sec}^2)$$

$$\text{and } L_1 = 10 \text{ unit.}$$

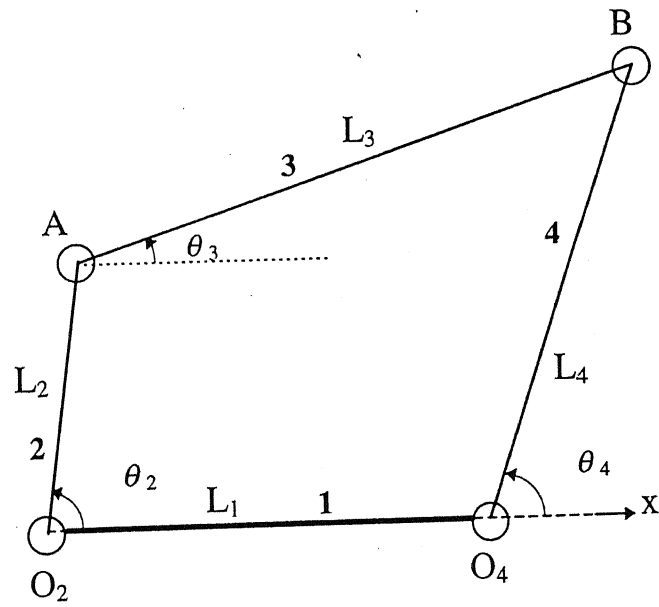
The results obtained, are

$$L_2 = 18.256 \text{ unit,}$$

$$L_3 = 22.419 \text{ unit}$$

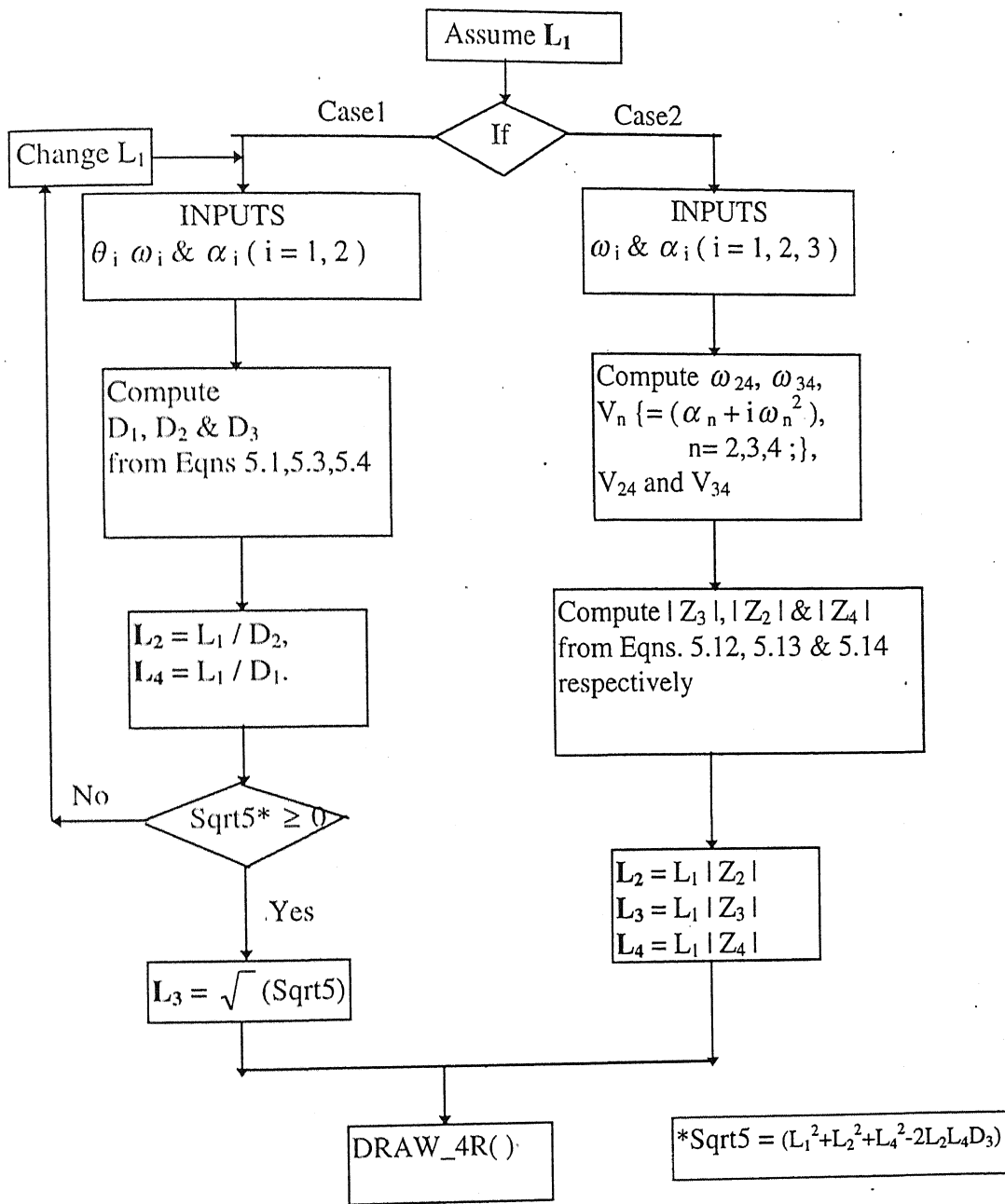
$$\text{and } L_4 = 5.2147 \text{ unit ;}$$

which give a mechanism of the crank-rocker type.

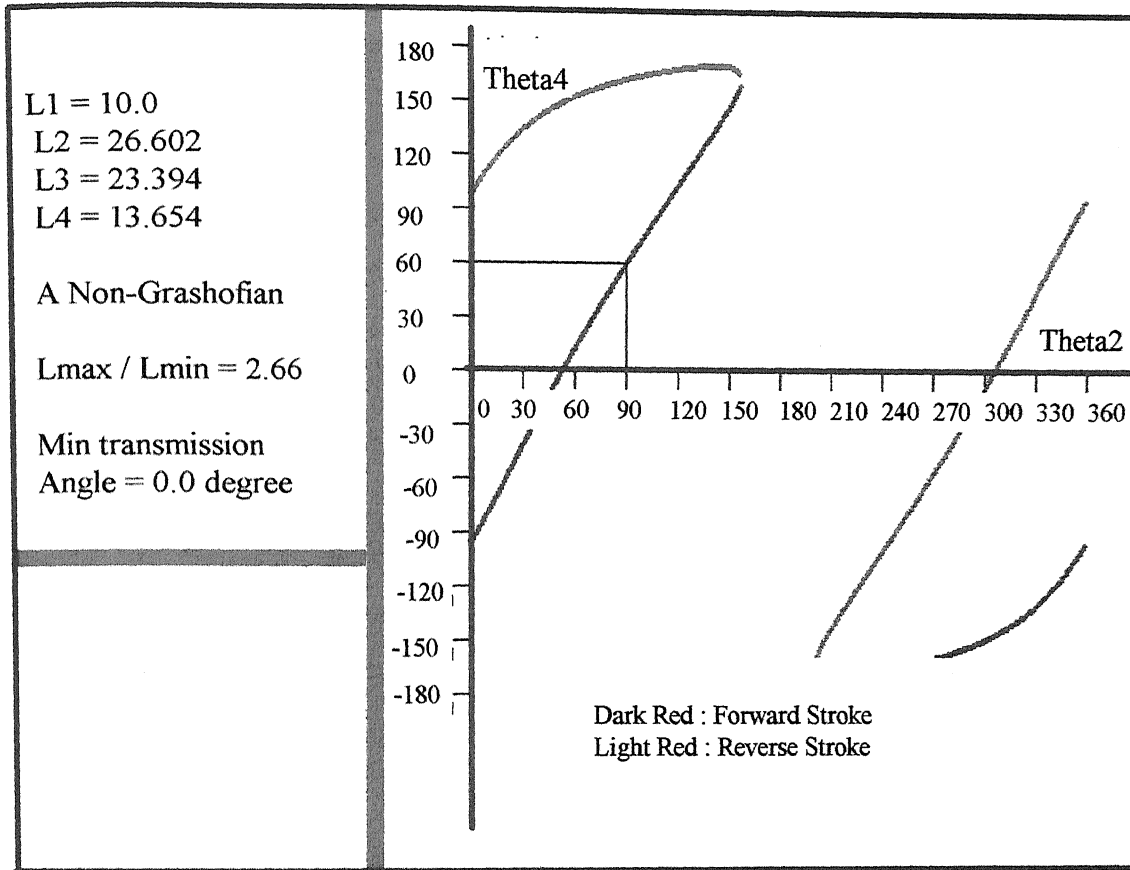


[Fig 5.1]

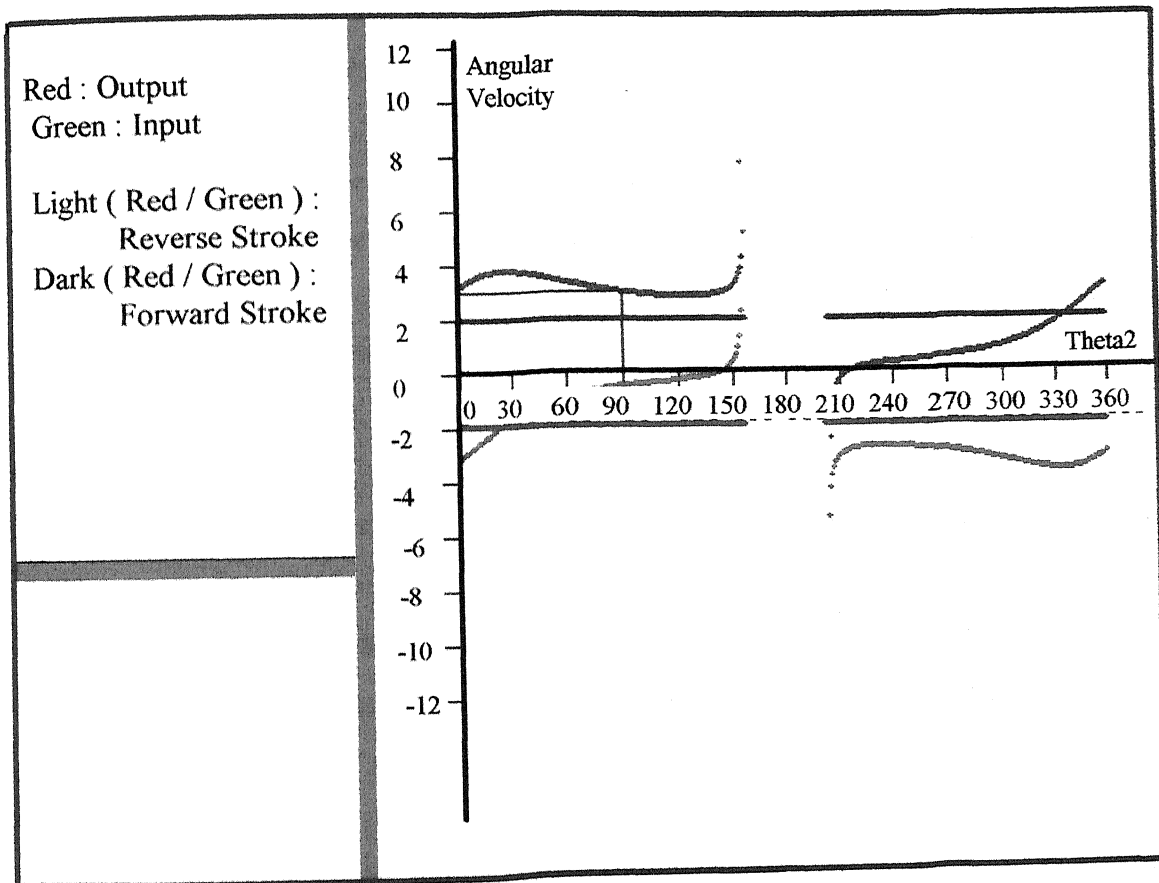
[PROBLEMS WITH KINEMATIC VARIABLES]



[Chart 5.1]



[Plot 5.1]



[Plot 5.2]

6. CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

6.1 CONCLUSIONS

On the basis of the results presented, the following conclusions can be drawn.

[i] The slider-crank mechanism for specified dead-center configurations under different cases (discussed in section 2.1) can be synthesized and visualized with animation. Having all the three inputs (H , Q_{RR} and one of the three parameters E , L_2 and L_3), the software checks the existence of a mechanism. Also an optimum design for prescribed values H and Q_{RR} can be obtained.

[ii] The synthesis of a crank-rocker mechanism for dead-center problems has been done without using the Volmer's nomogram. In comparison to the conventional nomogram approach, the new approach gives more accurate values of β_{opt} , $(\mu_{min})_{max}$ and hence the link-lengths for better transmission quality. The accuracy can also be controlled at the cost of execution load of the program as number of iterations will increase.

[iii] The overall optimization approach for the path generation problems can design 4R mechanisms for as many specified points as one wants, but the computational load will be increased. The present software also takes care of some constraints like the maximum-minimum link-lengths ratio and the compulsion for the designed mechanism to be a crank-rocker.

In the present software, an option to weight the error function (used for the least square technique) according to the errors has been provided. In this way the error can be further reduced. If there is any critical point through which the coupler point must pass, this approach will be useful but the errors at the other points may be increased.

Sometimes it is possible that we can have more than one solution, in that case one can choose the suitable one regarding other properties, like the transmission quality, the space requirement, the sensitivity to the tolerance, the quick-return ratio etc.. If the designer gets a Grashofian double-rocker as a solution, then using the property of cognates one can have crank-rocker solutions for the same coupler curve.

[iv] Overall optimization approach for the function generation problem also gives good results, but it may require some trials on the initial values. Here, one can also visualize the nature of the function on the screen. If instead of a function, a set of coordinated inputs and outputs are given, then one can guess the type of the standard functions (e.g. sinusoidal function) by visualizing the curve (θ_4 vs. θ_2).

[v] Regarding the two cusps problem, one can definitely obtain a crank-rocker mechanism with the proper selection of x_{O2} and x_{B1} (section 4.1). We can also visualize the movement of the coupler point from one cusp to other on the screen itself.

[vi] We can draw all the coupler curves of 4R and 3R-1P mechanisms. The coupler curves can be drawn in segments for some specified crank rotation, by which one can predict the relative velocity at different positions.

[vii] The problems with kinematic constraints (angular velocity, acceleration etc.) can also be solved for the two cases discussed in chapter 5.

6.2 SUGGESTIONS FOR FUTURE WORK

[i] All the problems, which we have discussed for four-link mechanisms can arise in the four-link module of a six-link and other complex mechanisms. In that situation, by synthesizing the four-link module, one may be able to synthesize the required mechanisms.

[ii] Besides the precision point and overall optimization approaches one can think about a *third approach* which includes the overall optimization and also takes care of some critical points. Some other constraints like the minimum transmission angle, tolerances etc. can also be included.

[iii] The weighted error function can also be used in the case of function generation problem and again one can consider the constraints which are mentioned in the path generation problem.

[iv] In the case of two cusps problem, since we can get more than one solution (depending upon the assumptions x_{O_2} and x_{B_1}), like dead-center problems of crank-rocker mechanisms one can maximize the minimum transmission angle to get a better crank-rocker mechanism.

[v] Like an atlas of coupler curves for crank-rocker mechanisms developed by Hrones and Nelson^[11], one can think of storing all the coupler curves for 4R and 3R-1P mechanisms, which can be treated as a quick guide for the designers.

[vi] One can design a 4R mechanism if kinematic conditions (Angular displacement, velocity and acceleration) for input and floating links are prescribed. This can be done by writing Freudenstein's equation in terms of θ_2 and θ_3 .

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APPENDIX

BASIC CLASSES OF C++ USED IN THE SOFTWARE

In developing a large and complicated software, classes are necessarily used. In the present software, the basic classes used, are

[i] **Revolute** - It contains all the necessary characteristics of revolute pairs.

The main member functions of this class are

- EnterData () ; This function asks for the link lengths through another class *LinkLength*.
- IfExist () ; It checks for the existence of a chain
- FixLink () ; After fixing a link chain becomes a mechanism.
- MaxMin () ; Finds the maximum and minimum lengths.
- IsGrashof () ; Checks for a grashofian mechanism.
- CalAngle () ; Calculates the output angle for each input angle, which is used in animation of 4R mechanism.
- CalRange () ; Calculates the range of the angular movement of any link of the mechanism.
- EnterOmega () ; Asks for the input angular velocity.
- EnterAcc () ; Asks for the input angular acceleration.

There is an inherited class *Prism* of the class *Revolute* for prismatic pairs.

[ii] **LinkLength** - It stores and returns the link length corresponding to a particular link.

[iii] **Range** - It stores and returns the range of linear or angular movement of any link of the mechanism.

[iv] **Animate** - Using this class animation of the mechanisms along with creating of menus and windows are done. The initial interaction with the user is made through a member function of this class. The main functions of this class are

- Draw_4R () ; Animates 4R mechanisms.
- Draw_3R1P () ; Animates 3R-1P mechanisms.
- Curve_4R () ; Plots coupler curve for 4R mechanisms.
- Curve_3R1P () ; Plots coupler curve for 3R-1P mechanisms.
- Cog4R () ; Animates the cognates of crank-rocker and double-rocker mechanisms.
- Display() ; Calculates the minimum transmission angle and display it along with the link lengths and types of mechanisms.
- PlotTh4 () ; It plots output angle vs input angles.
- PlotVelGraph () ; Plots input and output angular velocities corresponding to each input angular displacement.
- GiveAcc () ; It does acceleration analysis.

[v] **ScreenData** - This class interacts with the user and asks for the data's available through some member functions of class *synthesis1* to synthesis a mechanism for different types of problems.

[vi] **Angle** - It stores and returns the angular data's, which are used in synthesis of mechanisms.

[vii] **Point** - It stores and returns the linear data's or co-ordinates of points (required in path generation problems, cusp problems etc.), which are used in synthesis of mechanisms.

[viii] **Synthesis1** - This class is mainly used to compute the unknown design parameters for all types of synthesis problems which we have already discussed in this work. the main functions of this class are

- `AddPointData ()` ; With the help of this, the required linear inputs for different types of problems are asked.
- `AddAngleData ()` ; Same as above but this is for angular data 's.
- `Deadsol ()` ; Computes the unknown design parameters for dead centre problems.
- `SolveEq1 ()` ; For overall optimisation path generation problems
- `SolveEq2 ()` ;. For overall optimisation function generation problems.
- `Cuspsol ()` ; For two cusps problems.
- `Kinsol ()` ; For problems with kinematic constraints.

Thus in the case of synthesis of mechanisms, *ScreenData* interacts with the user and executes the member functions of *SynthesisI* (the member functions of *SynthesisI* execute the functions of classes *Angle and Point*) and transfer all the design parameters to class *Animate* for animation of the mechanism through `EnterData ()`.